

# Persian Curve Versus Monte Carlo Method

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**Abstract:** The Monte Carlo Simulation method is a powerful tool for solving problems including random variables. The basic idea to implement a Monte Carlo simulation is to first generate samples of random inputs from their assumed distribution functions and then perform a deterministic calculation on the generated random inputs, based on mathematical modeling of the system, to obtain output results. An early version of Monte Carlo simulation is the famous needle experiment. The idea of random experiment, have been used for solving many complex problems. Simulation based approaches have some disadvantages. Its implementation needs a massive use of computational resource and long calculation times. Moreover, providing linkage between input to the system and its output is difficult. Toward remedy, the phenomenon is considered as the change in the state of the system. Via logical reasoning, concise mathematics and using real world data, the output is related to the input via the Persian Curve. The Persian Curve provided a simple, cheap and exact solution to the problem. Consequently the Persian Curve is proposed as a replacement for the Monte Carlo Simulation. The validity of the work is verified via concise mathematics and comparison of the results with those of the others.

**Keyword:** Monte Carlo Simulation, Persian Curve, Change of State Philosophy, State Functions, Probability Density Function, Expected Value, Random Variable

## Introduction

The Monte Carlo Simulation (MCS) is a well-known statistical method which is used for analysis of phenomena with no clear deterministic solution, Fig. 1. Despite of its success in different field of human knowledge, the (MCS) is an expensive, time consuming and uncertain method (Badger, 1994; Veach, 1998; Bonate, 2001; Laosiritaworn, 2002; Lefebvre, 2007; Murray, 2007; Manohar, 2009; Wijesinghe, 2011; Du, 2012; Bolin, 2013; Cook *et al.*, 2013; Parkinson, 2013; Pollock, 2013; Romano, 2013; Zio, 2013; Goerdin, 2014; Poole, 2014; Rawlinson, 2015; Hahn, 2015; Wang, 2015; Hochuli, 2016; Sánchez, 2016; Zhao, 2016; Fadele, 2017; Feng, 2017; Mouawad, 2017; Schwarm, 2017a; 2017b; Haqiqat and Müller, 2018; Hou, 2018; Laengen, 2018; Albes, 2019; Huda, 2018; Pakyuz-Charrier, 2018; Unwin, 2018; Wang, 2018; Webster, 2019; Zhang, 2019; Mazhdrakov *et al.*, 2018; Corbella, 2019; de Freitas, 2019; Berg, 2019; Alamri, 2020; Apostolopoulou, 2019; Bhatia, 2020; Cumberworth,

2021; Ead, 2020; Ketron, 2020; Cosgrove, 2020; Dash, 2020; Debrot, 2020; Diniz, 2020; Eagle, 2020; Guijarro Gámez, 2020; Nilakanta, 2020; Welding, 2020; Wang, 2021; Sheridan-Methven, 2021; Sheridan-Methven, 2020). The Abdolrasoul Ranjbaran Team (ART) conducted an intensive research, for analysis of natural phenomena, in the past two decades in the Shiraz University, Iran (Ranjbaran *et al.*, 2008; Ranjbaran and Roustaa, 2009; Ranjbaran, 2010; Ranjbaran *et al.*, 2011; Ranjbaran, 2012; Ranjbaran and Roustaa, 2013a; 2013b; Ranjbaran *et al.*, 2013a; 2013b; Ranjbaran and Ranjbaran, 2014; Ranjbaran, 2015; Ranjbaran and Ranjbaran, 2016; 2017a; 2017b; 2017c; 2018; Ranjbaran *et al.*, 2020a-f; 2021a; 2021b). The result of her research is digested in the Persian Curve (PC), Fig. 2. The aim of the presented paper is to show that the (PC) is a good and logical alternative for the (MCS) in analysis of natural phenomena. The content of the paper is managed as follows. The basic principles of the (MCS) is expressed first. The study is continued with logical derivation of the (PC) basics.

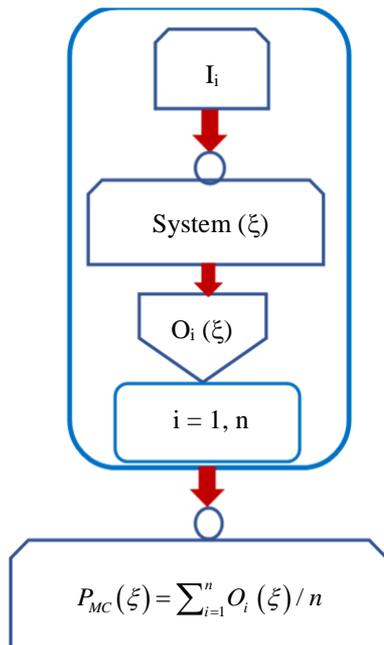


Fig. 1: Monte carlo simulation model

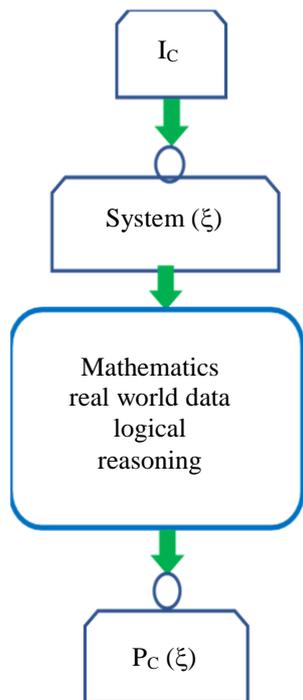


Fig. 2: Persian curve model

The work is continued with verification via analysis of fragility curve for structures, conclusions and list of references. In this study a phenomenon is considered as change in the state of a system. The state variable ( $\xi \in$

$[0,1]$ ) is defined as the lifetime or identification parameter of systems. Every system is expressed by its specific ( $\xi$ ). Consequently all derivations are finally expressed in terms of the ( $\xi$ ).

### The Monte Carlo Simulation

The Monte Carlo Simulation is a powerful tool for analysis of complex systems via including random variables (Badger, 1994; Veach, 1998; Bonate, 2001; Laosiritaworn, 2002; Lefebvre, 2007; Murray, 2007; Manohar, 2009; Wijesinghe, 2011; Du, 2012; Bolin, 2013; Cook *et al.*, 2013; Parkinson, 2013; Pollock, 2013; Romano, 2013; Zio, 2013; Goerdin, 2014; Poole, 2014; Rawlinson, 2015; Hahn, 2015; Wang, 2015; Hochuli, 2016; Sánchez, 2016; Zhao, 2016; Fadele, 2017; Feng, 2017; Mouawad, 2017; Schwarm, 2017a; 2017b; Haqiqat and Müller, 2018; Hou, 2018; Laengen, 2018; Albes, 2019; Huda, 2018; Pakyuz-Charrier, 2018; Unwin, 2018; Wang, 2018; Webster, 2019; Zhang, 2019; Mazhdrakov *et al.*, 2018; Corbella, 2019; de Freitas, 2019; Berg, 2019; Alamri, 2020; Apostolopoulou, 2019; Bhatia, 2020; Cumberworth, 2021; Ead, 2020; Ketron, 2020; Cosgrove, 2020; Dash, 2020; Debrot, 2020; Diniz, 2020; Eagle, 2020; Guijarro Gámez, 2020; Nilakanta, 2020; Welding, 2020; Wang, 2021; Sheridan-Methven, 2021; Sheridan-Methven, 2020). The basic idea to implement a Monte Carlo Simulation is to first generate samples of random inputs from their assumed distribution functions. Then perform a deterministic calculation on the generated random inputs, based on mathematical modeling of the system, to obtain numerical results. An early version of Monte Carlo Simulation is the famous needle experiment, performed by the French mathematician Comte de Buffon (1707-1788). Consider a plane with parallel lines distanced at ( $d$ ) and a needle, with a length ( $L < d$ ) that is randomly positioned on the plane. Note that: (1) The shortest distance from the needle center to the line ( $x$ ), is uniformly distributed over ( $0, d/2$ ). (2) The angle between the needle and a line ( $\theta$ ) also follows a uniform distribution over ( $0, \pi/2$ ). (3) the needle crosses a line when ( $x \leq L \sin \theta/2$ ). The two random variables ( $x$  and  $\theta$ ) are independent. Therefore, their joint probability density function is ( $f(x,\theta) = 4/\pi d$ ). Let ( $A$ ) denote the event that the needle lies across a line. The probability of ( $P(A) = 2L/\pi d$ ) is determined in Eq. (1) as follows:

$$P(A) = \int_0^{\pi/2} \left( \int_0^{L/2 \sin(\theta)} f(x,\theta) dx = \frac{L}{2} |\sin(\theta)| \frac{4}{\pi d} d\theta \right) \quad (1)$$

Buffon verified this probability by eventually throwing a needle on the plane with parallel lines. This experiment, reflects the basic idea of Monte Carlo Simulation. That is, performing the experiment ( $n$ ) times, if the needle crosses a line ( $m$ ) times, then the probability of ( $A$ ) is approximated in Eq. (2):

$$P(A) = m / n \tag{2}$$

As marked by Laplace in 1812, one can estimate the value of ( $\pi$ ) by conducting the Buffon's needle experiment and setting Eq. (1) equal to Eq. (2), in Eq. (3):

$$m / n = 2L / \pi d \rightarrow \pi = 2nL / md \tag{3}$$

In 1901, the Italian mathematician Lazzarini (Badger, 1994) conducted the needle experiments and obtained an estimate of the ( $\pi$ ) to six significant digits, as in Eq. (4):

$$(d; L; n; m) = (d; 5d / 6; 3480; 1808) \tag{4}$$

$$\pi = 3.14159292$$

This experiment offered such an impression that one can estimate a probability of an event or a random quantity via random simulation. In 1946, the physicist from Los Alamos were working on the distance likely to be traveled by the neutron in different materials under the Manhattan Project. They were unable to solve the problem using conventional deterministic mathematical methods. Then Stanislaw Ulam proposed an idea of using random experiments. This idea was subsequently developed by von Neumann, Metropolis and others to solve many complex problems in making the atomic bomb. Since the work was secret, the random experiment method required a code name. Metropolis suggested the name of Monte Carlo, which refers to the Monte Carlo Casino in Monaco where Ulam's uncle borrow money from relatives to gamble (Zhang, 2019). More scientific basis of the Monte Carlo simulation, is the strong law of numbers, which guarantees that the average of a set of independent and identically distributed random variables converges to the mean value with probability 1.

The strong law of large numbers is expressed as follows. For a sequence of statistically ( $n$ ) independent and identically distributed random variables ( $X_i, i = 1, n$ ) with a mean value of ( $\mu$ ), then the average of the variables converges to ( $\mu$ ) with probability 1, as in Eq. (5):

$$\sum_{i=1}^n O_i / n \rightarrow \mu \text{ as } n \rightarrow \infty \tag{5}$$

Monte Carlo simulation model is shown in Fig. 1. According to the strong law of large numbers, the average converges to certain value as ( $n \rightarrow \infty$ ). Equation (5), for a system with the characteristics function ( $h(\xi, x)$ ) and the random system output ( $x = (x_1, \dots, x_n)$ ) with the probability distribution ( $f_o(x)$ ) is expressed in Eq. (6) for the system ( $\xi$ ):

$$P_{MC}(\xi) = \int_{-\infty}^{+\infty} h(\xi, O) f_o(O) dO \tag{6}$$

The idea of random experiment, have been used for solving many complex problems (Badger, 1994; Veach, 1998; Bonate, 2001; Laosiritaworn, 2002; Lefebvre, 2007; Murray, 2007; Manohar, 2009; Wijesinghe, 2011; Du, 2012; Bolin, 2013; Cook *et al.*, 2013; Parkinson, 2013; Pollock, 2013; Romano, 2013; Zio, 2013; Goerdin, 2014; Poole, 2014; Rawlinson, 2015; Hahn, 2015; Wang, 2015; Hochuli, 2016; Sánchez, 2016; Zhao, 2016; Fadele, 2017; Feng, 2017; Mouawad, 2017; Schwarm, 2017a; 2017b; Haqiqat and Müller, 2018; Hou, 2018; Laengen, 2018; Albes, 2019; Huda, 2018; Pakyuz-Charrier, 2018; Unwin, 2018; Wang, 2018; Webster, 2019; Zhang, 2019; Mazhdrakov *et al.*, 2018; Corbella, 2019; de Freitas, 2019; Berg, 2019; Alamri, 2020; Apostolopoulou, 2019; Bhatia, 2020; Cumberworth, 2021; Ead, 2020; Ketron, 2020; Cosgrove, 2020; Dash, 2020; Debrot, 2020; Diniz, 2020; Eagle, 2020; Guijarro Gámez, 2020; Nilakanta, 2020; Welding, 2020; Wang, 2021; Sheridan-Methven, 2021; Sheridan-Methven, 2020). Simulation based approaches have some disadvantages. Its implementation needs a massive use of computational resource and long calculation times. Moreover, providing linkage between input to the system and its output via a closed form solution is difficult. In view of Eq. (6) the aim is determination of the ( $P_{MC}$ ), which is only a function of ( $\xi$ )! There is no sign of the method of solution in it. It appears that using the (MCS) for this job is the worst method that could be selected. Despite of this much of efforts, since the infinity is out of reach then the solution by the (MCS) always contains epistemic uncertainty which is added to the aleatory one. Toward remedy, the Abdolrasoul Ranjbaran Team (ART), conducted an extensive research toward analysis of change in the systems, called the Change of State Philosophy (CSP). The result of her research is the Persian Curve (PC) which is expressed in the next section. As will be observed in the next section, the (ART) investigations, via logical reasoning, concluded directly to the Persian Curve ( $P_C(\xi)$ ), which clearly can be used in place of ( $P_{MC}(\xi)$ )! As will be seen, the Persian Curve is equal to the expected value of the system output (obtained by stochastics methods including the (MCS) for  $n \rightarrow \infty$ ) with probability 1. All methods in human knowledge are approximate (since  $n \ll \infty$ ) as compared to the Persian Curve.

### The Persian Curve Basics

For a change in a system (e.g., structure) the survived capacity and the fled capacity are important (Ranjbaran *et al.*, 2008; Ranjbaran and Rousta, 2009; Ranjbaran, 2010; Ranjbaran *et al.*, 2011; Ranjbaran, 2012; Ranjbaran and

Rousta, 2013a; 2013b; Ranjbaran *et al.*, 2013a; 2013b; Ranjbaran and Ranjbaran, 2014; Ranjbaran, 2015; Ranjbaran and Ranjbaran, 2016; 2017a; 2017b; 2017c; 2018; Ranjbaran *et al.*, 2020a-f; 2021a; 2021b). The capacity ( $k_s$ ), called system stiffness, is defined as a positive entity which has a finite specified value at the beginning of the lifetime (origin) and reduces to zero at the end of systems lifetime (destination). The inverse of the system stiffness is called system flexibility ( $f_s$ ). The concept of stiffness and flexibility are used for better understanding! They are general and are not necessarily those used in the structural mechanics! For a system with a given lifetime, the survived stiffness ( $k_{SS} = k_s - k_C$ ) and the survived flexibility ( $f_{SS} = f_s + f_C$ ) are obviously inverse of each other, where ( $k_C$ ) is the change stiffness and ( $f_C$ ) is the change flexibility. This obvious relation is defined in Eq. (7) and shown in Fig. 3 and is used for reliable analysis of changing systems as follows:

$$(k_s - k_C)(f_s + f_C) = 1 \quad (7)$$

Note that although the ( $k_s$  and  $f_s$ ) are known at the origin, the ( $k_C$  and  $f_C$ ) at each point along the lifetime should be determined. This is done as follows.

There is only one Eq. (7) at hand for solution. Therefore, for the time being, let the ( $k_s$ ,  $f_s$  and  $f_C$ ) as known and solve Eq. (7) for ( $k_{SS}$  and  $k_C$ ) in Eq. (8). Where (-;-) denotes a vector:

$$(k_{SS}; k_C) = (S_R; F_R) k_s \quad (8)$$

In Eq. (8) the phenomenon functions (the survived function ( $S_R$ ) and the fled function ( $F_R$ )) are defined in Eq. (9):

$$(S_R; F_R) = (f_s; f_C) / (f_s + f_C) \quad (9)$$

Note that the phenomenon functions are ratios in a unit interval. Since construction of functionals in terms of two functions is not possible, then the phenomenon functions are customized to the state functions as follows.

For ( $f_s = 1$ ), the phenomenon functions are renamed as the state functions (collection of the Origin function (O) and the destination function (D)) and the state Ratio (R) in Eq. (10):

$$(f_s; f_C; S_R; F_R) = (1; R; O; D) \quad (10)$$

In view of Eq. (10), the state functions are defined as ratios in unit intervals in Eq. (11):

$$(O; D) = (1; R) / (1 + R) \quad (11)$$

Eq. (11) may be rewritten as boundary value problems in Eq. (12), in which (min and max) denote (minimum and maximum) respectively:

$$(O; D) = \begin{cases} (\max; \min) = (1; 0), @ R = 0 \\ (\min; \max) = (0; 1), @ R = \infty \end{cases} \quad (12)$$

Since, from Eq. (11) the state ratio ( $R = D/O$ ) is defined in a semi-infinite interval ( $R \in [0, \infty]$ ) and it is a function, then it cannot be used as an independent variable, Fig. 4! Toward remedy, a state variable in a unit interval ( $\xi \in [0, 1]$ ), with a zero value at the origin ( $\xi = 0$ ) and a one value at the destination ( $\xi = 1$ ) is defined. In view of the artifice used and in terms of the state variable, Eq. (12) is rewritten as Eq. (13):

$$(O; D) = \begin{cases} (\max; \min) = (1; 0), @ \xi = 0 \\ (\min; \max) = (0; 1), @ \xi = 1 \end{cases} \quad (13)$$

Make use of the expertise of the authors the state functions are defined as explicit functions of the ( $\xi$ ), in Eq. (14) where each function is an averages of a polynomial- and a trigonometric-function, as in Eq. (15), Fig. 5:

$$\begin{aligned} 4O &= (2 + 1 - 6\xi^2 + 4\xi^3 + \cos \pi\xi) \\ 4D &= (2 - 1 + 6\xi^2 - 4\xi^3 - \cos \pi\xi) \end{aligned} \quad (14)$$

$$\begin{aligned} 4O &= (1 + 1 - 6\xi^2 + 4\xi^3) + (1 + \cos \pi\xi) \\ 4D &= (1 - 1 + 6\xi^2 - 4\xi^3) + (1 - \cos \pi\xi) \end{aligned} \quad (15)$$

At this stage, attention is paid to enhance the construction of the phenomenon functions. Via the definition of the ( $k_{SS}$  and  $f_{SS}$ ) in Eq. (7) and concept of crack compliance ( $f_C$ ) in fracture mechanics (Anderson, 2017), the authors detected a fact that, the ( $f_C$ ) is directly proportional to the ( $k_s$ ). This detection is called, the Persian Principle of Change (PPC). In view of the (PPC) the ( $f_C$ ) is innovatively defined in Eq. (16):

$$(f_C / R; f_C) = k_s / 1; k_s D / O \quad (16)$$

Substitution of Eq. (16) into Eq. (9), concluded into the generalized definition for the phenomenon functions in Eq. (17).

$$(S_R; F_R) = (O; k_s^2 D) / (O + k_s^2 D) \quad (17)$$

The ( $k_s$ ) can not be directly determined from the real world data (e.g., reliable test results). Toward better definition and providing the condition for using real world data, Eq. (17) is rewritten in Eq. (18) in terms of two positive control parameters (a<sub>M</sub>) and (b) (Ranjbaran *et al.*, 2020b). Flexibility for translation and rotation of phenomenon functions in the (1 × 1) working space, which lets the experts to enforce their will, is provided by the form of phenomenon functions in Eq. (18) and selection of two control parameters from reliable real world data:

$$(S_R; F_R) = \frac{(O^b; a_M D^b)}{O^b + a_M D^b} \quad (18)$$

To this end the system identification flag (state variable) and the mathematical basis for determination of the capacity is proposed, separately, in abstract form. Consequently the

work is certain and universal, in a sense that it is independent of geometry, size, material property and etc. Therefore, it equally applies to all natural phenomena as change in systems. The system identification flag and the basic mathematical formulation are connected via the Persian Curve as follows.

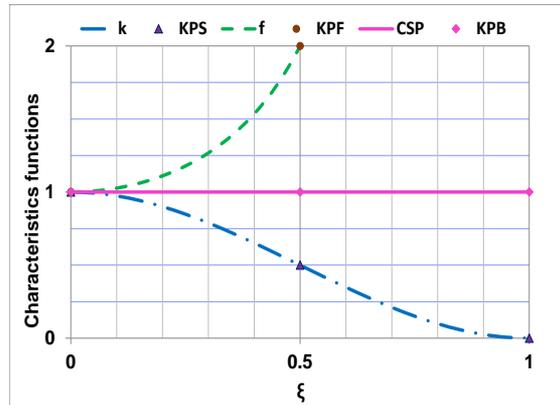


Fig. 3: Change of state philosophy basic equation

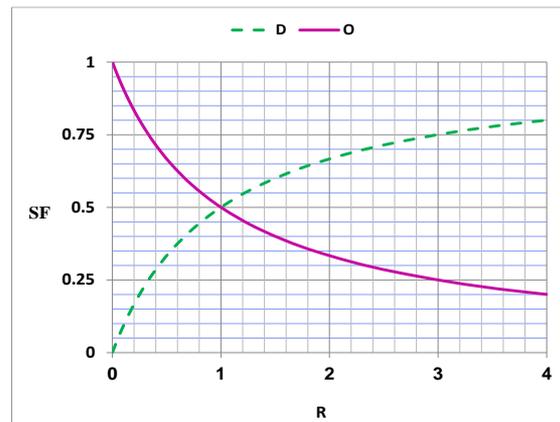


Fig. 4: State Functions versus the state ratio

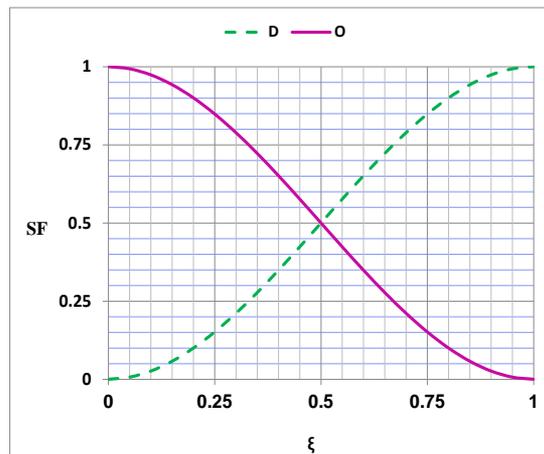


Fig. 5: State Functions versus the state variable

### Persian Curve Birth

The basic formulation for determination of capacity ratio is derived independent of real world data. In order to be able to apply the proposed formulation to natural phenomena (change in real systems), it must be connected to the system identification flag. This is done via construction of Persian Curve as follows. The phenomenon functions are mapped onto the real world data. The ratios at the beginning ( $P_O$ ) and at the termination ( $P_T$ ) of the phenomenon is selected from the reliable real world data, Fig. 6. For real natural phenomenon the ( $S_R$ ) is renamed as Shiraz curve ( $P_S$ ) and the ( $F_R$ ) is renamed as Fasa curve ( $P_F$ ) and collection of the two is called Persian Curve (PC) as defined in Eq. (19). Note that Eq. (19) for ( $P_O = 1$  and  $P_T = 0$ ) and ( $P_O = 0$  and  $P_T = 1$ ) converges to ( $S_R$ ) and ( $F_R$ ) in Eq. (18) respectively:

$$P_C = \frac{P_O O^b + P_T a_M D^b}{O^b + a_M D^b} \tag{19}$$

In comply with common practice in the literature, the derivative of phenomenon functions with respect to ( $\xi$ ) is called the Zahedan curve ( $P_Z$ ) as defined in Eq. (20), in which ( $D^{(1)}$ ) is derivative of ( $D$ ) with respect to ( $\xi$ ):

$$(F_R; S_R) = \int_{(0;\xi)}^{(\xi;1)} P_Z dx \tag{20}$$

$$P_Z = \frac{b a_M D^{b-1} O^{b-1} D^{(1)}}{(O^b + a_M D^b)^2}$$

Note that, the ( $P_Z$ ) corresponds to the Probability Density Function (PDF) in the literature. To this end the formulation is complete. What remained is determination of the control parameters ( $a_M$  and  $b$ ) for a real world data. This is done as follows. To prepare for simple and user friendly analysis, Eq. (19) is rewritten in Eq. (21), in which ( $P_C$ ) is coordinate of a point on real world data along the lifetime:

$$a_M = \frac{a_C}{(D/O)^b}, a_C = \frac{P_C - P_O}{P_T - P_C} \tag{21}$$

The aim is determination of ( $a_M$  and  $b$ ) from nonlinear Eq. (21). Toward the aim, the coordinates of the middle point (M) with ( $D_M/O_M = 1$ ) and the next point (N) (a point between  $O$ ,  $M$  and  $T$ ) are used in Eq. (21) to determine ( $a_M$  and  $b$ ) from Eq. (22). Eq. (22) introduces the simplest method for analysis of nonlinear Eq. (21) and is the best tool for construction of a smooth curve to go through the key points ( $O$ ,  $N$ ,  $M$  and  $T$ ) and with the extremum points at the ends, which is made with the expert will:

$$(a_{N;M}; b) = \left( \frac{P_{N;M} - P_O}{P_T - P_{N;M}}; \frac{\log(a_N/a_M)}{\log(D_N/O_N)} \right) \tag{22}$$

The Key Points (KPS) are defined as the origin point (O), the middle point (M), the termination point (T) and the next point (N) (a point between the other three), in Eq. (23) and shown in Fig. 6 for both of increasing and decreasing data:

KPS	O	N	M	T
$\xi$	0.0	$\xi_N$	0.5	1.0
P	$P_O$	$P_N$	$P_M$	$P_T$

(23)

### Unified Persian Curve

Extensive review of literature (ANSI/AISC 360-16, 2016; AS 4100, 1998; Gardner, 2002; Ranawaka, 2006; Heva, 2009; Dolamune Kankanamge, 2010; Keerthan, 2010; Haidarali, 2011; Shahbazian, 2013; Cheng, 2015; Kucukler *et al.*, 2015; Nguyen, 2017; Qiu, 2017; Szalai 2017; Rasmussen, 2017; Ye, 2018; Imran *et al.*, 2018), regarding real world data for failure of systems (Structures), led to the unified control parameters in Eq. (24) and the unified Persian curves in Eq. (25):

$$(a_M; b) = (2; 1) \tag{24}$$

$$(P_{SU}; P_{FU}; P_{ZU}) = \left( \frac{1-0.8D}{1+D}; \frac{1.8D}{1+D}; \frac{2D^{(1)}}{(1+D)^2} \right) \tag{25}$$

In view of Eq. (25) the unified key points coordinates are defined in Eq. (26). The Persian Curve model is shown in Fig. 2, in a form to be comparable with the Monte Carlo model in Fig. 1:

PC	O	N	M	T
$\lambda$	0.000	0.750	1.500	3.000
$\xi$	0.000	0.250	0.500	1.000
$P_{FU}$	0.000	0.237	0.600	0.900
$P_{SU}$	1.000	0.763	0.400	0.100
$P_{ZU}$	0.000	1.686	1.364	0.000

(26)

### Comparison of the Persian Curve and the Monte Carlo Simulation

Comparison of Eq. (6) and Eq. (19), that is the Monte Carlo curve (PMC) and the Persian Curve (PC), shown that both are system specific functions. Albeit the former is expensive and contains epistemic uncertainty, while the latter is cheap, simple and free of epistemic uncertainty. The idea behind the two are shown

graphically in Fig. 1 and 2. respectively. More comparison of the results are included as follows.

The Persian curve is defined in terms of an abstract lifetime in a unit interval ( $\xi \in [0, 1]$ ). In real phenomena the lifetime is selected in a truncated interval ( $\lambda \in [\lambda_o, \lambda_T]$ ), where ( $\lambda_o$ ) is the origin lifetime and ( $\lambda_T$ ) is the termination lifetime. In order to use the Persian Curve for real world data, the ( $\lambda$ ) should be mapped onto ( $\xi$ ) as in Eq. (27):

$$\lambda = (1 - \xi)\lambda_o + \xi\lambda_T$$

$$\xi = (\lambda - \lambda_o) / (\lambda_T - \lambda_o) \tag{27}$$

In comply with the literature, the results of the Persian Curve is compared with that of the Monte Carlo Method in the following example.

Example 1: Iervolino *et al.* (2016; Baltzopoulos *et al.*, 2017), developed a fragility curve for a six-story reinforced concrete moment resisting Frame (BF), via using the SPO2FRAG software. Compare the (BF) with the Persian-Fasa-curve as the proposed fragility function. Note that the SPO2FRAG is developed based on the Monte Carlo Method.

Solution: Via hundreds of thousands of expensive non-linear incremental dynamic analyses, the required data is prepared and the Fragility curve (BF) (a log-normal curve fitted on the data) is developed, which is scanned and shown in Fig. 7. The selected Key Points coordinates (KPF) and the corresponding control parameters ( $a_M$ ) and (b) are expressed in Eq. (28) and Eq. (29):

KPF	O	N	M	T
$\xi$	0.00	0.25	0.50	1.00
P	0.00	0.237	0.60	0.90

(28)

$\lambda$	$\xi$	$a_M$	b
[0,3]	[0,1]	2.00	1.00

(29)

The unified Persian-Fasa-Curve (PFU), the unified Persian-Shiraz-curve (PSU) and the unified Persian-Zahedan-curve (PZU) from Eq. (25) are also shown in Fig. 7. Total agreement of the results in general and at the key points (KPF) in specific, is used as a flag for verification of the validity of the unified fragility curve. This close agreement is a powerful reason for recommending the (PC) as a replacement for the (MCS)

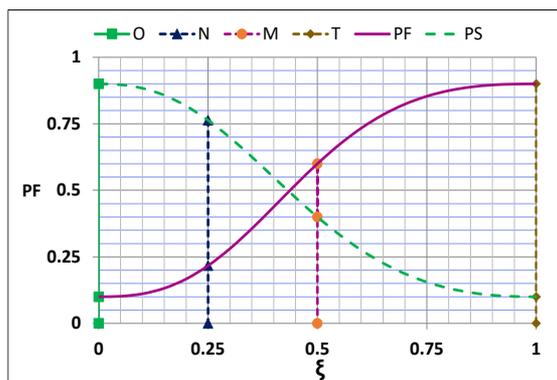


Fig. 6: Key points on persian-shiraz- and persian-fasa-curves

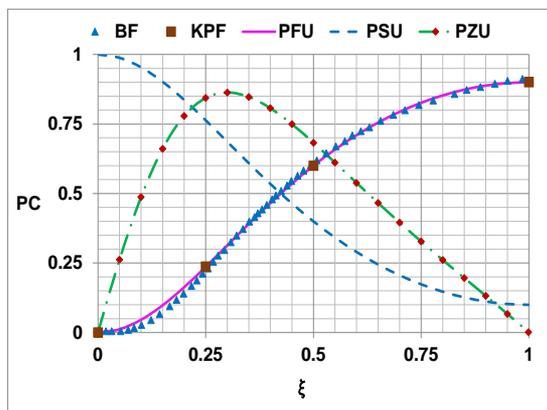


Fig. 7: Comparison of the (BF) and the persian curves

## Conclusion

The following conclusions are obtained from the present study:

- The Persian Curve is equal to the expected value of output in (MCS) with probability 1
- The Persian Curve is an excellent replacement for the Monte Carlo Simulation
- The Persian Curve is the best method for analysis of real world data
- The problems that were solved by the Monte Carlo Simulation is recommended to be resolved with the Persian Curve
- Replacement of the Monte Carlo Simulation by the Persian Curve is a revolution in human knowledge

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## Author's Contributions

**Abdolrasoul Ranjbaran:** Designed and contributed in writing the first draft and later corrections of the manuscript.

**Mohammad Ranjbaran:** Contributed in literature review, derivation of equations and engaged in the research team discussions.

**Fatema Ranjbaran:** Contributed in numerical analysis and engaged in the research team discussions.

## Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved with declaration of no conflict of interest.

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## List of Symbols

$A$ :	Event of needle crossing line
$a_M$ :	Control parameter at $M$
$a_N$ :	Control parameter at $N$
$b$ :	Control parameter (power)
$CSP$ :	Change of State Philosophy
$D$ :	Destination function
$d$ :	Distance between parallel lines
$E$ :	Initial modulus
$f(x, \theta)$ :	Joint probability density function
$F$ :	Force
$F_y$ :	Yield limit
$f_S$ :	System flexibility
$f_C$ :	Change flexibility
$F_S$ :	Dimensioned flexibility
$f_{SS}$ :	Survived flexibility
$F_R$ :	Failure function
$(F_R \text{ and } S_R)$ :	Phenomenon functions
$f_O(O)$ :	Probability density function
$h(\xi, \theta)$ :	System characteristics function
$I_i$ :	Random input variable
$k_S$ :	System stiffness
$k_C$ :	Change stiffness
$k_{SS}$ :	Survived stiffness
$K_{PF}$ :	Key points on Failure curve
$K_{PS}$ :	Key points on Survive curve
$K_{PB}$ :	Key points on CSP curve
$L$ :	Effective length
$L$ :	Needle length
$\lambda$ :	Lifetime parameter
$\lambda = \frac{L}{r} \sqrt{\frac{F_y}{\pi^2 E}}$ :	Relative slenderness ratio
$\lambda_O$ :	Lifetime origin
$\lambda_T$ :	Lifetime termination (end)
$\mu$ :	Average of random output
$m$ :	Number of crossing event
$M$ :	Middle point
$M_{CS}$ :	Mont Carlo Simulation
$n$ :	Number of random variables
$n$ :	Number of needle throw
$N$ :	Next point
$O$ :	Origin (start) point.
$O$ :	Origin function
$O_i$ :	Random output variable
$O_D$ :	Ordered designation
$P(A)$ :	Probability of A
$P_C$ :	Persian curve (s)
$P_O$ :	Origin point ordinate
$P_N$ :	Next point ordinate
$P_{MC}$ :	Mont Carlo Simulation function.
$P_M$ :	Middle point ordinate
$P_T$ :	End point ordinate
$P_{FU}$ :	Unified Persian-Failure function

$P_{SU}$ :	Unified Persian-Survive function	$S_R$ :	Survive function
$P_{ZU}$ :	Unified Persian-distribution function	$SF = (D \text{ and } O)$ :	State functions
$P_Z$ :	Persian-Distribution function	$T$ :	Termination (end) point
$P_F$ :	Persian-Failure function	$\theta$ :	Angle between line and needle
$P_S$ :	Persian-Survive function	$\xi$ :	State variable
$PC = (PF \text{ and } PS)$ :	Persian curves	$\psi$ :	Displacement
$r$ :	Effective radius of gyration	$x$ :	Working parameter
$R$ :	State ratio		