

MIXED CONVECTION BOUNDARY LAYER FLOW ON A VERTICAL SURFACE IN A POROUS MEDIUM SATURATED BY A NANOFLUID WITH SUCTION OR INJECTION

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Received 2013-02-07, Revised 2013-03-13; Accepted 2013-05-10

ABSTRACT

An analysis of the steady mixed convection boundary layer flow past a vertical permeable surface embedded in a porous medium saturated by a nanofluid is performed in this study. Numerical solutions of the similarity equations are obtained using the shooting method. Three types of metallic or nonmetallic nanoparticles, namely Copper (Cu), Alumina (Al₂O₃) and Titania (TiO₂) are considered by using a water-based fluid to investigate the effect of the solid volume fraction or nanoparticle volume fraction parameter ϕ of the nanofluid. The numerical results of the skin friction coefficient and the velocity profiles are presented and discussed. It is found that the imposition of suction is to increase the velocity profiles and to delay the separation of boundary layer, while the injection parameter decreases the velocity profiles. On the other hand, the range of solutions for the injection case is largest for Al₂O₃ nanoparticles and smallest for Cu nanoparticles.

Keywords: Boundary Layer, Nanofluid, Mixed Convection, Permeable Surface, Porous Medium

1. INTRODUCTION

The problem of convection in a porous medium provides one of the basic scenarios for heat transfer theory and thus is of considerable theoretical and practical interest and has been extensively studied. Excellent reviews of the topic can be found in the books by Nield and Bejan (2006); Pop and Ingham (2001); Ingham and Pop (2005) and Vadasz (2008). The most basic problem for natural or free convection in a porous medium past a vertical flat plate was first studied by Cheng and Minkowycz (1997). There are several numerical studies on the mixed convection in a porous media and we mention here those by Harris *et al.* (2009); Rosali *et al.* (2011); Imran *et al.* (2012) and Mukhopadhyay (2012). On the other hand, nanofluids are engineered by suspending nanoparticles with average size below 100 nm in traditional

heat transfer fluids such as water, oil and ethylene glycol. Fluids such as water, oil and ethylene glycol are poor heat transfer fluids, since the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Choi and Guarino (1995) showed that the addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluids up to approximately two times. Therefore, the effective thermal conductivity of nanofluids is expected to enhance heat transfer compared to the conventional heat transfer liquids.

Some numerical and experimental studies on the forced and natural convection using nanofluids related with differentially heated enclosures have been considered by Jou and Tzeng (2006); Tiwari and Das (2007); Abu-Nada (2008); Oztop and Abu-Nada (2008);

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Muthamilselvan *et al.* (2010) and Ghasemi and Aminossadati (2010). Nield and Kuznetsov (2009) have studied the Cheng and Minkowycz's problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid taking into account the combined effects of heat and mass transfer in the presence of Brownian motion and thermophoresis as proposed by Buongiorno (2006). Later, Kuznetsov and Nield (2010) examined the natural convective heat transfer in the boundary layer flow of a nanofluid past a vertical flat plate embedded in a porous medium. The steady boundary layer flow of a nanofluid past a stretching sheet using Buongiorno (2006) nanofluid model has been discussed by Khan and Pop (2010). The model they used for the nanofluid incorporates the effects of Brownian motion and thermophoresis and found solution which depends on the Prandtl number Pr , Lewis number Le , Brownian motion number Nb and thermophoresis number Nt . Also, Bachok *et al.* (2010) used the Buongiorno (2006) nanofluid equation model for the steady boundary layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. They assumed that the plate is moving in the same or opposite directions to the free stream to define resulting system of nonlinear ordinary differential equations.

Recently, Ahmad and Pop (2010) considered the steady mixed convection boundary layer flow over a vertical flat plate embedded in a porous medium filled with a nanofluid using the nanofluid equation model proposed by Tiwari and Das (2007). The review papers by Arifin *et al.* (2011; 2012a); Yasin *et al.* (2012) and Arifin *et al.* (2012b) present excellent collections of published papers on nanofluids using model proposed by Tiwari and Das (2007). Therefore, the present investigation deals with the steady mixed convection boundary layer flow past a permeable vertical flat plate embedded in a porous medium saturated with a nanofluid. This study extends the papers by Ahmad and Pop (2010) to the case of permeable surface. Results are presented in tables and figures showing the effects of the constant suction or injection parameters.

2. MATERIALS AND METHODS

2.1. Problem Formulation

Consider the steady mixed convection boundary layer flow past a vertical semi-infinite plate embedded in a porous medium filled with a nanofluid. It is assumed that the free stream velocity and the ambient temperature (far

flow from the plate) are U_∞ and T_∞ , respectively. It is also assumed that the temperature of the plate is T_w , where $T_w > T_\infty$ corresponds to a heated plate (assisting flow) and $T_w < T_\infty$ corresponds to a cooled plate (opposing flow). It is also assumed that the convecting fluid and the porous medium are in local thermodynamic equilibrium, the viscous dissipation is neglected, the physical properties of the fluid except the density are constant and that the Boussinesq approximation holds. Following the nanofluid equation model proposed by Tiwari and Das (2007) along with the Boussinesq and boundary layer approximations, it is easy to show that the steady boundary layer equations of the present problem are (Ahmad and Pop, 2010):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\mu_{nf}}{\mu_f} \frac{\partial u}{\partial y} = \frac{gK[\phi\rho_s\beta_s + (1-\phi)\rho_f\beta_f]}{\mu_f} \frac{\partial T}{\partial y} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Subject to the boundary conditions:

$$\begin{aligned} v &= v_w, \quad T = T_w(x) \text{ at } y = 0 \\ u &\rightarrow \infty, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \tag{4}$$

Integrating Equation 2 with the boundary conditions Equation 4, it becomes:

$$\frac{\mu_{nf}}{\mu_f} u = \frac{\mu_{nf}}{\mu_f} U_\infty + \frac{gK[\phi\rho_s\beta_s + (1-\phi)\rho_f\beta_f]}{\mu_f} (T - T_\infty) \tag{5}$$

Here, x and y are the Cartesian coordinates measured along the plate and normal to it, respectively, u and v are the velocity components along x and y axes, respectively, T is the temperature of the nanofluids, g is the acceleration due to gravity, $v_w(x)$ is the mass transfer velocity with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection, ϕ is the nanoparticle volume fraction, μ_f is the dynamic viscosity of the base fluid, β_f and β_s are the coefficients of thermal expansion of the fluid and of the solid, respectively, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively, μ_{nf} is the viscosity of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid, which are given by Oztop and Abu-Nada (2008):

$$\begin{aligned} \mu_{nf} &= \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \end{aligned} \tag{6}$$

where, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid as expressed by Khanafer *et al.* (2003) and Abu-Nada (2008) The viscosity of the nanofluid μ_{nf} can be approximated as the viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles. The effective thermal conductivity of the nanofluid k_{nf} is approximated by the Maxwell-Garnett's model, which is found to be appropriate for studying the heat transfer enhancement using nanofluids (Abu-Nada, 2008) We now look for similarity solutions of Equation 3 and 5 subject to the boundary conditions (4) of the following form:

$$\begin{aligned} \psi &= \alpha_f (2Pe_x)^{1/2} f(\eta), \quad \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \\ \eta &= \frac{(Pe_x)^{1/2} y}{x\sqrt{2}} \end{aligned} \tag{7}$$

where, $Pe_x = U_\infty X/a_f$ is the local Peclet number for the porous medium and ψ is the stream function, which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = \partial\psi/\partial x$. Thus, we have Equation (8):

$$\begin{aligned} u &= U_\infty f'(\eta), \\ v &= -\frac{1}{2} \frac{\alpha_f (2Pe_x)^{1/2}}{x} [f(\eta) - \eta f'(\eta)] \end{aligned} \tag{8}$$

where, primes denote differentiation with respect to η . In order that Equation 1 to 3 subject to the boundary conditions 4 admit a similarity solution, we have to consider that $v_w(X)$ has the following expression Equation (9):

$$v_w(x) = -\frac{1}{2} \frac{\alpha_f (2Pe_x)^{1/2}}{x} f_0 \tag{9}$$

where, f_0 is the constant mass transfer parameter with $f_0 > 0$ for suction and $f_0 < 0$ for injection.

Substituting Equation 6 and 7 into Equation 3 and 5, we obtain the following system of ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5}} f'' = \frac{1}{(1-\phi)^{2.5}} + \left[(1-\phi) + \phi \left(\frac{\rho_s}{\rho_f} \right) \left(\frac{\beta_s}{\beta_f} \right) \right] \lambda \theta \tag{10}$$

$$\frac{\frac{k_{nf}}{k_f}}{(1-\phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}} \theta'' + f\theta' = 0 \tag{11}$$

Along with the boundary conditions:

$$f(0) = f_0, \quad \theta(0) = 1, \quad f'(\infty) \rightarrow 1, \quad \theta(\infty) \rightarrow 0$$

where, λ is the constant mixed convection parameter, which is defined as Equation 12:

$$\lambda = \frac{Ra_x}{Pe_x} \tag{12}$$

With $Ra_x = \rho_f g K \beta_f (T_w - T_\infty) x / \mu_f a_f$ being the local Rayleigh number for a porous medium. It is worth mentioning that $\lambda > 0$ corresponds to an assisting flow (heated plate), $\lambda < 0$ corresponds to an opposing flow (cooled plate) and $\lambda = 0$ corresponds to the forced convection flow. Further, Equation 10 and 11 can be combined to give single equation:

$$\frac{k_{nf} / k_f}{(1-\phi) + \phi(\rho C_p)_s / (\rho C_p)_f} f'' + ff'' = 0 \tag{13}$$

Subject to the boundary condition Equation (14):

$$\begin{aligned} f(0) &= f_0, \\ \frac{1}{(1-\phi)^{2.5}} f'(0) &= \frac{1}{(1-\phi)^{2.5}} \\ + \left[(1-\phi) + \phi \left(\frac{\rho_s}{\rho_f} \right) \left(\frac{\beta_s}{\beta_f} \right) \right] \lambda f'(\infty) &= 1 \end{aligned} \tag{14}$$

The physical quantity of interest is the skin friction coefficient C_f , which is defined as:

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2} \tag{15}$$

where τ_w is the skin friction or the shear stress at the surface of the plate, which is given by:

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{16}$$

Substituting Equation 7 into Equation 15 and 16, we obtain Equation 17:

$$(2Pe_x)^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(\theta) \quad (17)$$

3. RESULTS AND DISCUSSION

The nonlinear ordinary differential Equation 13 subject to the boundary conditions 14 was solved numerically using the shooting method. This well-known technique is an iterative algorithm which attempts to identify appropriate initial conditions for a related Initial Value Problem (IVP) that provides the solution to the original Boundary Value Problem (BVP). The shooting method is based on MAPLE “dsolve” command and MAPLE implementation, “shoot” (Meade *et al.*, 1996). The results are given to carry out a parametric study showing the influences of the non-dimensional parameters, namely the mixed convection parameter λ and the constant suction/injection parameter f_0 . Following Oztop and Abu-Nada (2008) we have considered the range of nanoparticles volume fraction ϕ as $0 \leq \phi \leq 0.2$. The thermophysical properties of fluid and nanoparticles (Cu, Al₂O₃, TiO₂) used in this study are given in **Table 1**. In order to validate the accuracy of the numerical method used, the present results for the skin friction coefficient $f''(0)$ when $\phi = 0.1$ and $\phi = 0.2$ for Cu nanoparticles and various values of λ are compared with those of Ahmad and Pop (2010), as shown in **Table 2 and 3**, respectively. It is clearly seen that the comparison shows very good agreement. **Table 2 and 3** also illustrate the influence of the suction and injection parameter $f_0 = 0.5$ (suction), 0 (impermeable) and -0.5 (injection) for Cu nanoparticles and various values of λ . It should be noticed that the results given in the parentheses are the second (dual) solutions. The results indicate that the imposition of suction ($f_0 > 0$) at the surface has the tendency to increase the skin friction coefficient $f''(0)$ but for the case of surface injection ($f_0 < 0$), the skin friction coefficient $f''(0)$ decreases. Based on our computations, the critical values of λ (say λ_c) are presented in **Table 4**, which show a favorable agreement with the previous investigations for the case of impermeable surface ($f''(0)$).

Figure 1 shows the variation of the skin friction coefficient $(2Pe_x)^{1/2} C_f$ with λ for different types of nanoparticles (Cu, Al₂O₃, TiO₂) when $\phi = 0.1$. This figure shows that it is possible to get dual solutions of the similarity Equation 13 subjected to boundary conditions 14 for the opposing flow case ($\lambda < 0$) with upper and lower branch solutions. Dual solutions exist for $\lambda_c < \lambda < 0$, a unique solution exists for $\lambda = \lambda_c < 0$ and no solutions exist for $\lambda = \lambda_c < 0$, where λ_c is the critical value of λ for which the solution exists. As in similar physical situations, we postulate that the upper branch solutions are physically stable and occur in practice, whilst the lower branch solutions are not physically realizable. This postulate can be verified by performing a stability analysis but this is beyond the scope of the present paper. On the other hand, it is also shown in **Fig. 1** that suction ($f_0 > 0$) delays separation compared to the impermeable surface or injection ($f_0 < 0$) cases. This is true for all the three nanoparticles (Cu, Al₂O₃, TiO₂) considered.

The variation of the skin friction coefficient $(2Pe_x)^{1/2} C_f$ with suction/injection parameter f_0 when $\phi = 0.1$ and $\lambda = -1.6$ are presented in **Fig. 2**. This figure supports the dual nature of the solutions to the boundary-value problem (14) and (15). For this value of λ , there is a critical value f_{0c} of f_0 , at which there is a saddle-node bifurcation, with two solutions for $f_0 > f_{0c}$ and no solutions for $f_0 < f_{0c} < 0$. This indicates that injection (having $f_0 < 0$) limits the existence of solutions, whereas no such limit appears for suction ($f_0 > 0$), with both branches of solutions continuing to large values of $f_0 > 0$ (suction). Based on our computations, the values of f_{0c} are $f_{0c} = -0.515151$ for Al₂O₃, $f_{0c} = -0.49825$ for TiO₂ and $f_{0c} = -0.33955$ for Cu. This shows that the range of solutions for the injection ($f_0 < 0$) case is largest for Al₂O₃ nanoparticles and smallest for Cu nanoparticles. **Figure 3-5** show the velocity profiles for the first (upper branch) and second (lower branch) solutions when $\phi = 0.1$ for different types of nanoparticles, namely Cu, Al₂O₃, TiO₂ respectively. The dashed line refers to the second (lower branch) solution and these solution profiles prove the existence of dual solutions. These figures also show that the suction parameter increases the velocity profiles and the injection parameter decreases the velocity profiles. Finally, it is worth mentioning that all the velocity profiles presented in **Fig. 3-5** satisfy the far field boundary conditions (15) asymptotically and thus, support the validity of the dual solutions obtained.

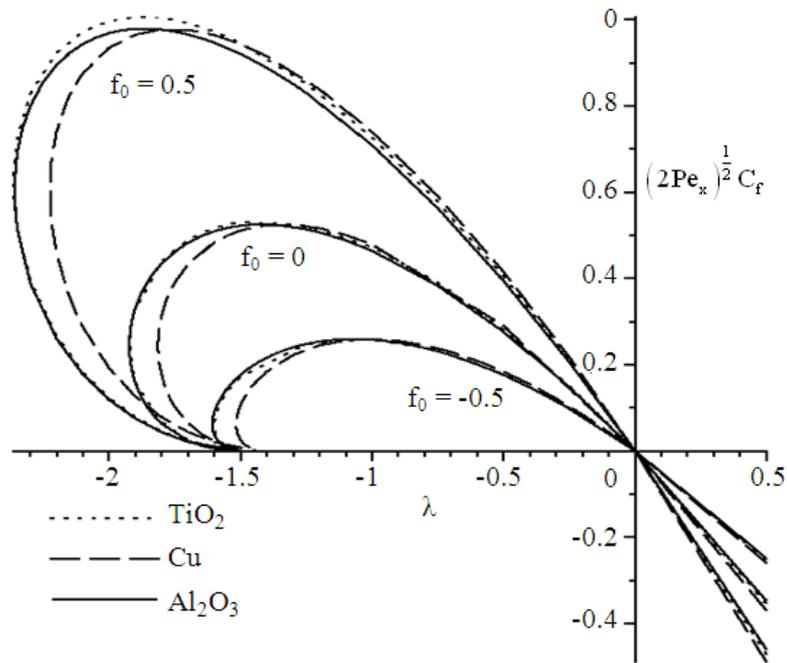


Fig. 1. Variation of the skin friction coefficient $(2Pe)^{1/2}C_f$ with λ for different types of nanoparticles when $\phi = 0.1$ and various values of f_0

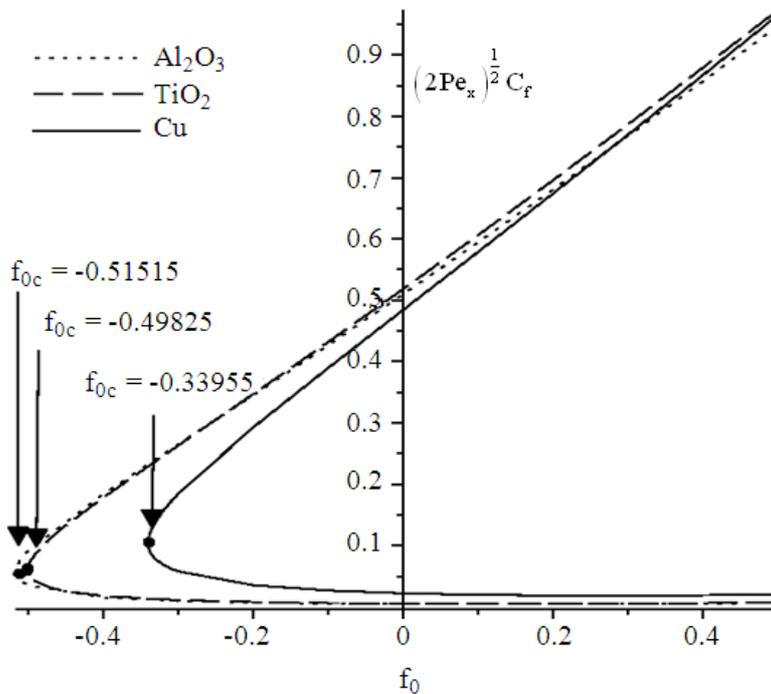


Fig. 2. Variation of the skin friction coefficient $(2Pe)^{1/2}C_f$ with f_0 for different types of nanoparticles when $\phi = 0.1$ and $\lambda = -1.6$

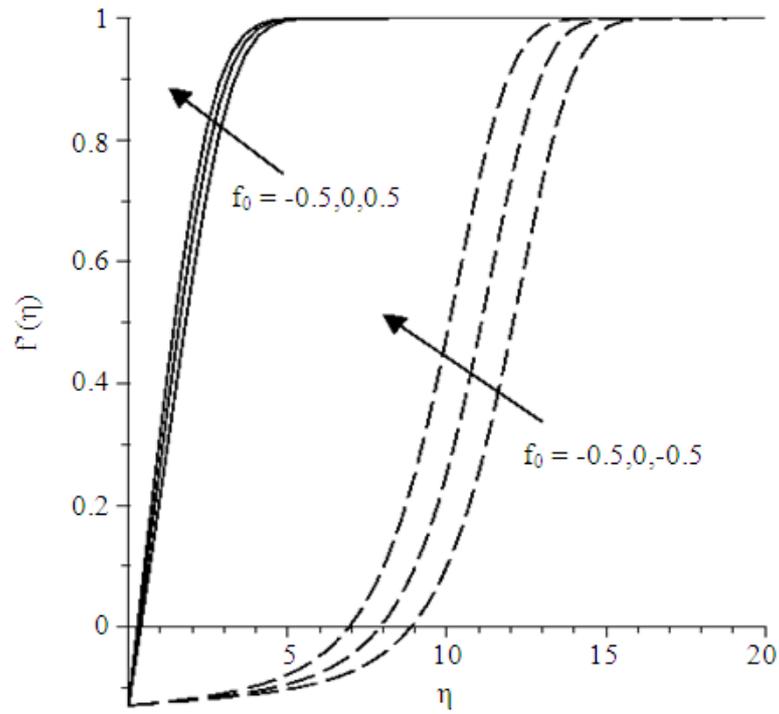


Fig. 3. Velocity profiles $f'(\eta)$ for Cu nanoparticles when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0 .

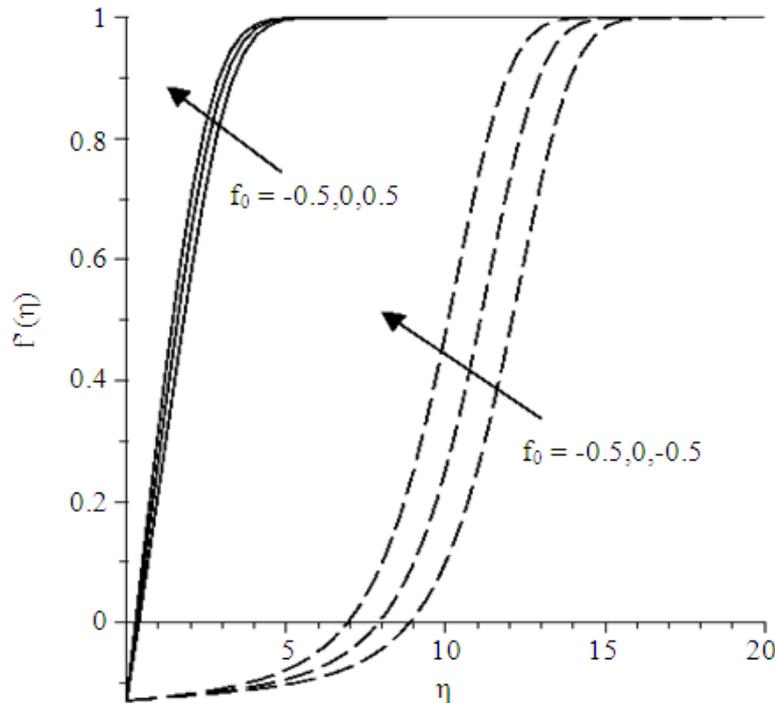


Fig. 4. Velocity profiles $f'(\eta)$ for Al_2O_3 nanoparticles when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0 .

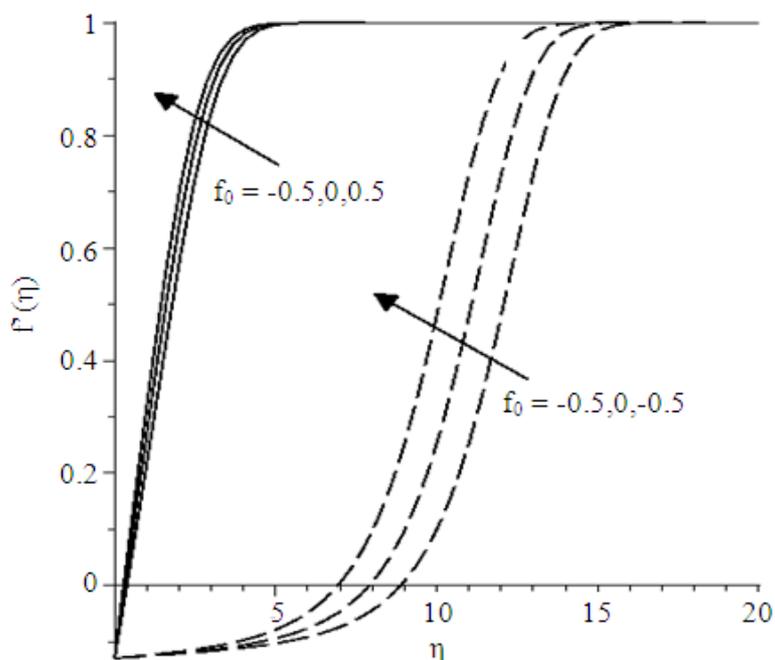


Fig. 5. Velocity profiles $F'(\eta)$ for TiO_2 nanoparticles when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0

Table 1. Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada, 2008)

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p (J/kg K)	4179.000	385	765	686.2000
ρ (Kg/m ³)	997.100	8933	3970	4250.0000
K (W/mK)	0.613	400	40	8.9538

Table 2. Values of $f'(0)$ for Cu nanoparticles when $\phi = 0.1$

λ	Ahmad and Pop (2010)		Present	
	$f_0 = 0$		$f_0 = -0.5$	$f_0 = 0.5$
-1.45	0.39852(0.00070)		0.11885(0.00345)	0.71481
-1.50	0.39263(0.00333)		0.08218(0.01966)	0.72505
-1.51693			0.04746	
-1.55	0.38391(0.00864)			0.73381 (0.00633)
-1.60	0.37176 (0.01733)			0.74101 (0.01572)
-1.65	0.35523 (0.03309)			0.74651 (0.02394)
-1.70	0.33259 (0.04957)			0.75017 (0.03387)
-1.75	0.30004 (0.07864)			0.75180 (0.04587)
-1.80	0.24204 (0.13322)			0.75117 (0.06014)
-1.81433			0.18714	
-1.85				0.75117 (0.06014)
-1.90				0.74188 (0.09678)
-1.95				0.73230 (0.12006)
-2.00				0.71850 (0.14760)
-2.10				0.67524 (0.22106)
-2.2259				0.46446

Table 3. Values of $f'(0)$ for Cu nanoparticles when $\phi = 0.2$

λ	Ahmad and Pop (2010)	Present		
	$f_0 = 0$	$f_0 = -0.5$	$f_0 = 0$	$f_0 = 0.5$
-1.75	0.34746	0.17741	0.34746	0.54393
-2.00	0.34528 (0.00051)	0.13506 (0.00159)	0.34528	0.57983
-2.14460		0.05192		
-2.20	0.32401 (0.01328)		0.32409 (0.01328)	0.59687
-2.25	0.31477 (0.02044)		0.31476 (0.02044)	0.59919
-2.30	0.30313 (0.02991)		0.30311 (0.02991)	0.60065 (0.02208)
-2.35	0.28848 (0.04240)		0.28845 (0.04240)	0.60115 (0.02483)
-2.40	0.26954 (0.05920)		0.26949 (0.05920)	0.60065 (0.03472)
-2.45	0.24337 (0.08332)		0.24321 (0.08322)	0.59903 (0.04443)
-2.50	0.19693 (0.12878)		0.19559 (0.12878)	0.59618 (0.05501)
-2.50987			0.16214	
-2.60				0.58620 (0.07921)
-2.70				0.56902 (0.11070)
-2.80				0.54140 (0.15266)
-2.9				0.49506 (0.21336)
-2.99337				0.36095

Table 4. Variation of λ_c for different types of nanoparticles (Cu, Al₂O₃, TiO₂) for $\phi = 0.1$

	Ahmad and Pop (2010)	Present	
		λ_c	λ_c
Cu	$f_0 = -0.5$		-1.51690
	$f_0 = 0$	-1.814	-1.81433
	$f_0 = 0.5$		-2.22590
Al ₂ O ₃	$f_0 = -0.5$		-1.60800
	$f_0 = 0$	-1.923	-1.92351
	$f_0 = 0.5$		-2.36157
TiO ₂	$f_0 = -0.5$		-1.59900
	$f_0 = 0$	-1.918	-1.91902
	$f_0 = 0.5$		-2.36301

4. CONCLUSION

The present work deals with the steady mixed convection boundary layer flow past a vertical surface embedded in a porous medium saturated by a nanofluid as considered by Ahmad and Pop (2010). We have extended the previous work by taking into consideration the effects of suction or injection with permeable surface. Further, the governing equations are transformed into ordinary differential equations and are then solved numerically using the shooting method. The effects of the suction or injection parameter, the mixed convection parameter and the nanoparticle volume fraction parameter on the flow and heat transfer characteristics are studied. In general, imposition of suction is to increase the velocity profiles and to delay the separation of boundary layer, while the injection parameter decreases the velocity

profiles. On the other hand, the range of solutions for the injection case is largest for Al₂O₃ nanoparticles and smallest for Cu nanoparticles.

5. ACKNOWLEDGEMENT

The researchers gratefully acknowledge the financial support received in the form of a RUGS research grant from Universiti Putra Malaysia.

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