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Fractional Calculus Theoretical Evolution for Radiation Quantities

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Abstract: Problem statement: Radiation dosimetry features depend on semi-empirical formulas that lack a strong mathematical framework. This is due to the fact that the microscopic radiation interaction with matter includes energy losses that have never been described properly in quantum mechanics, which deals with conserved energy systems. **Approach:** Using the recent theory of the quantization of nonconservative systems using fractional calculus. **Results:** Most important charged particle interaction features and consequences like energy loss, stopping power, range, absorbed dose and radiotoxicity are frame-worked mathematically. **Conclusion:** The results manifest a good agreement with experimental and semi-empirical results.

Key words: Fractional calculus, quantization of nonconservative systems, interaction of radiation with matter, energy loss, absorbed dose, stopping power, dissipative medium

INTRODUCTION

When a charged particle incidents on matter, it interacts with the electrons and the nucleus of every atom in the lane it passes through. Most of these interactions convey only tiny fractions of the charged-particle's kinetic energy. So the particle seems to be as losing its kinetic energy gradually in a friction-like practice, termed "continuous slowing-down approximation". Because of the huge number of interactions occured to each charged particle in slowing down, its path length tends to approach the expectation value that would be observed as an average for a very large number of identical particles. This expectation value is called the range. The expectation value of the speed of energy loss per unit of path length x of a charged particle of kinetic energy in a medium of atomic number Z, is called the stopping power, which was derived for the first time classically by Bohr, with all the fundamental derivation features are classical or semiclassical in origin(Ajlouni, 2010).

A full quantum-mechanical treatment is needed to obtain exact results. A number of people, including Bethe and Bloch, have discussed the quantum-mechanical derivation of the energy loss from the point view of the inelastic-scattering cross section to justify quantum-mechanically Bohr's formula for energy loss. Formulas are found semiempirically with several correction coefficients. The Bethe-Bloch formula remains the starting point (Ajlouni, 2010, Foschini *et al*., 2002).

Fractional calculus launches as a very useful tool could be used to express the dissipation which is associated with, almost, all physical processes in nature (Ajlouni, 2011), by having new operators, new criteria and new set of subordination theorems could be obtained with some earlier results and standard methods (Thongwan *et al*., 2011; AL-Ghonaiem *et al*., 2010; Cansee *et al*., 2010). The new complexified dynamics guides to a new dynamics which may differ totally from the classical mechanics cardinally and may bring new appealing consequences. Some additional interesting results are explored and discussed in some details (Rami, 2011).

In this study we formulate mathematically the most important charged particle interaction features like energy loss, stopping power, range and absorbed dose, depending on the well frame-worked theory of quantization of nonconservative systems using fractional calculus and mainly on the canonical quantization of a system of free particles in a dissipative medium.

MATERIALS AND METHODS

According to the theory of the quantization of nonconservative systems (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011), the Hamiltonian can be written as follows (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

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$$H = \sum_{i=0}^{N-1} \frac{d^{s(i+1)-s(i)}}{d(t-b)^{s(i+1)-s(i)}} q_{r,s(i)} p_{r,s(i)} - L \ ,0 \le i \le N-1$$
$$= \sum_{i=0}^{N-1} q_{r,s(i+1)} p_{r,s(i)} - L \quad (1)$$

And the Schrödinger equation reads (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$H\Psi = ih \frac{\partial}{\partial t}\Psi \quad (2)$$

Consider a free particle moving in a dissipative medium where dissipation is proportional to velocity (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011), i.e:

$$F = -\gamma q_1 \quad (3)$$

where, γ being a positive constant. The potential related to this dissipation is (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$U = \frac{i\gamma}{2} q_{1/2}^2 \quad (4)$$

The Lagrangian is (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$L = \frac{1}{2} m q_1^2 - \frac{i\gamma}{2} q_{1/2}^2 \quad (5)$$

Where:

$$q_0 = x, \quad q_1 = \frac{dx}{dt}, \quad q_{1/2} = \frac{d^{1/2}x}{d(t-b)^{1/2}} \quad (6)$$

The canonical momenta are (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$p_0 = \frac{\partial L}{\partial q_{1/2}} + i \frac{d^{1/2}}{d(t-a)^{1/2}} \frac{\partial L}{\partial q_1}$$
$$= i\gamma\, q_{1/2} + im q_{3/2} \quad (7)$$

And:

$$p_{1/2} = \frac{\partial L}{\partial q_1} = m q_1 \quad (8)$$

Making use of Eq. 1, we have for the Hamiltonian (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$H = \frac{(p_{1/2})^2}{2m} + q_{1/2} p_0 + \frac{\gamma}{2i} q_{1/2}^2 \quad (9)$$

Here p_0 and p1/2 are the canonical conjugate momenta to q_0 and q1/2, respectively.

Schrdinger's equation reads (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$ih \frac{\partial}{\partial t}\Psi = \left[-\frac{h^2}{2m} \frac{\partial^2}{\partial q_{1/2}^2} + \frac{h}{i} q_{1/2} \frac{\partial}{\partial q_0} + \frac{1}{2i}\gamma q_{1/2}^2 \right]\Psi \quad (10)$$

Which is Schrdinger's equation for a dissipated free particle, has the following solution (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$\Psi_n = A H_n \left[\left(\frac{m\gamma}{i\hbar^2} \right)^{1/4} q_{1/2} \right] \exp$$
$$\frac{-\left(\frac{m\gamma}{i\hbar^2} \right)^{1/2} q_{1/2}^2}{2} \exp \frac{i}{\hbar} E_x \frac{q_0}{q_{1/2}} \exp \frac{-i}{\hbar} E_0 t \quad (11)$$

where, H_n are Hermite polynomials.

RESULTS

Energy loss: The dissipative effects result almost exclusively from the electric field of the electrons inside the matter. Many processes may occur during the particle's passage; however, what concerns us most is the overall energy loss of the particle in a slab of matter of thickness x>R, R, being the range of the charged particle in matter.

The dissipative (Coulomb) force is assumed to be proportional to the velocity (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$F = \frac{zeZe}{r^2} = -\gamma q_1 \quad (12)$$

Since the particle is losing its kinetic energy gradually in a friction-like process, as has already been mentioned. The kinetic energy gained by one electron and lost by the charged particle is:

$$\frac{p_e^2}{2m_0} = \frac{2z^2e^4}{m_0 b^2 v^2} \quad (13)$$

This energy is lost by the charged particle as dissipation and appears as a reduction in its velocity; so we can write (Ajlouni *et al*., 2012; Ajlouni, 2010; 2011):

$$\frac{2z^2e^4}{m_0b^2v^2} = \frac{i\gamma}{2}q_{1/2}^2 = \frac{i\gamma}{2}y^2 \quad (14)$$

Depending on Ajlouni Theory presented in Eq. 2-17, the Energy expectation value is:

$$\langle E\rangle = \int_{-\infty}^{\infty}\int_{x=0}^{x}\Psi^{*}H\Psi dydx' \quad (15)$$

Which could be written as:

$$\langle E\rangle = \int_{x'=0}^{x}\int_{-\infty}^{\infty} H_n \exp\left(\frac{-\alpha^2y^2}{2}\right)\left(\exp\frac{-i}{\hbar}E_x\frac{x'}{y}\right)\left[\alpha^2\frac{iE_xx'}{\hbar y} - \left(\frac{E_xx'}{\hbar y^2}\right)^2 + \frac{iE_xx'}{\hbar y^3}\right]\times H_n\exp\left(\frac{-\alpha^2y^2}{2}\right)\exp\left(\frac{i}{\hbar}E_x\frac{x}{y}\right)dydx' \quad (16)$$

Using mathematical identities:

$$\langle E\rangle = bx^3 - ax^2 - cx \quad (17)$$

So that:

$$\frac{d\langle E\rangle}{dx} = gx^2 - fx - c \quad (18)$$

where, a, b, c, f and g are constants.

DISCUSSION

Stopping power and linear energy transfer: Several terms are used to describe the changes in energy of a particle and the absorbing medium. The stopping power, S, is defined as the loss of energy from a particle over a path length dx:

$$S = -\frac{d\langle E\rangle}{dx} \quad (19)$$

Very often the term mass stopping power is used which is defined as:

$$S = -\frac{1}{\rho}\frac{d\langle E\rangle}{dx} \quad (20)$$

where, ρ is the density of the material. Mass stopping power is divided into two components: collision and radiative stopping power.

From Eq. 18 and 19 we get:

$$S = -\frac{d\langle E\rangle}{dx} = -gx^2 + fx + c \quad (21)$$

Range of charged particle inside matter: The range or distance that a heavy charged particle will travel in a material can be obtained by integrating the energy loss rate along the path of the ion. In the approximation that the ion follows a straight-line trajectory, then the range for a given kinetic energy, R (E), would be given by the integral:

$$R = \int_0^{E_0}\left[\frac{d\langle E\rangle}{dx}\right]^{-1} dE \quad (22)$$

where, the function $d\langle E\rangle/dx$ is the appropriate function for the ion in the material. There are two difficulties in applying this simple integral, the ions will suffer a different number of collisions with atomic electrons and, more importantly, the ions will undergo some scattering from the Coulomb fields of the atomic nuclei. The multiple Coulomb scattering leads to an effect that the ion's trajectory is not straight but rather is made up from a series of straight line segments causing the effect of range straggling which is indicated by the Gaussian distribution of ranges. For the practicing nuclear scientists, range-energy tables or relationships are among the most commonly used tools.

By means of Eq. 18-22 takes a simple form:

$$R = \frac{E_0}{gx^2 - fx - c} \quad (23)$$

With agreement with the experimental fact that, heavy charged particles penetrate uniformly into matter with essentially no attenuation in intensity until they are nearly at rest; at this point the intensity of moving ions rapidly drops to zero as represented in Fig. 1. It is very clear from Eq. 23 that the rang depends on E_0, thus a mono-energetic charged particle beam will have the same Rvalue.

Absorbed dose: Charged particles deposit their energy in the medium through which they propagate. The total energy deposited per unit mass (of the medium) is called the dose. It has units of Gy (Gray):

$$1Gy = 1J/kg = 6.25\times10^{12} MeV/kg \quad (24)$$

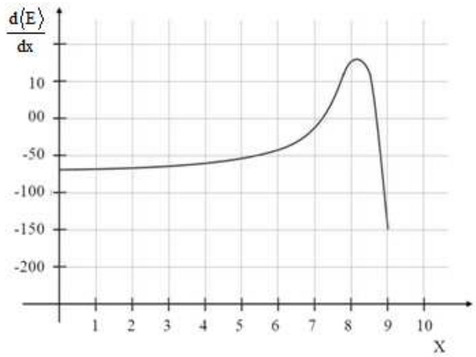

Fig. 1: Average charged particle energy loss as a function of distance through matter

The dose clearly refers to a flux of particles over a specified period of time or to a specified event (e.g., a nuclear accident) and given by:

$$D = \phi \frac{-d\langle E \rangle}{\rho dx} \quad (25)$$

Using Eq. 18-25 becomes:

$$D = -\varphi\left(gx^2 - fx - c\right) \quad (26)$$

Which is very similar to Eq. 21.

Radiotoxicity: The radiotoxicity, RT, associated with a given nuclide is the effective dose (in Sievert, Sv, which is the unit of dose equivalents to biological systems, especially people) resulting from ingesting or inhaling an activity A (in Bq, the unit of radioactivity, where 1 Becquerel (Bq)×1 disintegration = s). It can be written as:

$$RT = \frac{A\omega_R \langle E \rangle \tau_{eff}}{m_{eff}} \quad (27)$$

Where:

$\langle E \rangle$ = The mean energy per decay deposited in the body

ω_R = The associated risk factor

Depending on the nucleus in question, the effective retention time ι_{eff} can be the nuclear mean life for short-lived nuclei, the biological retention time for elements that are eliminated from the body, or the lifetime of the organism itself for long-lived nuclei that can be permanently attached to the body parts, e.g., ^{239}Pu in bones. The effective mass of the organism, m_{eff}, is the body mass if one calculates the full-body dose or the mass of the organ in which the radioactive material is attached if one wishes to calculate the dose received by that organ. Referring to Eq. 18-27 is written as:

$$RT = \frac{A\omega_R (bx^3 - ax^2 - cx)\tau_{eff}}{m_{eff}} \quad (28)$$

The radiotoxicity, as Eq. 28 manifests, is larger and more effective than the absorbed dose since we are speaking about the equivalent dose in body due to total amount of energy deposited by the charged particle resulting from ingesting or inhaling an activity. This explains the fact that radiotoxicity is mostly discussed in the context of nuclear accidents and nuclear-waste storage.

CONCLUSION

In this study we treat the problem of energy loss as a friction-like problem and give no consideration to detailed processes occurring during the interaction of charged particles with matter. The energy loss of charge particles is expressed as a function of the penetration depth. We found that, most of the energy is deposited near the stopping point. This is very similar to the fact that charged particles deliver a significant fraction of their kinetic energy at the end of their range makes charged particles useful for radiation therapy.

The charged particle range in matter is expressed also as a function of depth. We find that mono-energetic charged particle beam will have the same range inside matter, in agreement with the experimental facts that the intensity of charged particle beam stays constant for some distance and drop drastically to zero at the end of the range.

The formula of the absorbed dose versus the distance traveled inside matter has been derived, performing large agreement with experimental results.

For small x-values the particle is very fast and the interaction probability so the energy loss is small and only few ion pairs are formed, thus the absorbed dose, is very low; for larger x- values, as the charged particle advances inside matter, the particle becomes slower and the interaction probability is then very high. So the energy loss is large and many ion pairs are formed as a result of the passage of the charged particle, thus the absorbed dose, is very high.

The radiotoxicity is larger and more effective than the absorbed dose, since the equivalent dose in body due to total amount of energy deposited by the charged particle resulting from ingesting or inhaling an activity.

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