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Approximate Solutions of a Higher Order MHD Flow of a Uniformly Stretched Vertical Permeable Surface in the Presence of Heat Generation/Absorption which Resulted from a Quadratic Reaction

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Abstract: Higher order MHD flow of a uniformity stretched vertical permeable surface in the presence of heat generation/absorption which resulted from a quadratic reaction was studied. The resulting coupled nonlinear partial differential equations is solved by asymptotic expansion. It was discovered that the maximum value of velocity occurs in the body of the fluid close to the surface not at the surface. The effect of heat generated on the velocity profile is more pronounced than that of heat absorption.

Key words: MHD flow, quadratic reaction, heat generation/absorption, velocity, Prandtl number, permeable surface

INTRODUCTION

Adequate knowledge of flow and heat and mass transfer in the boundary layer induced by a surface moving with a uniform or non-uniform velocity in a quiescent ambient fluid is important in several manufacturing processes in industry which include boundary layer along material handling conveyers, the extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath (Chamkha, 2003). Also this type of problem arises in glass blowing continuous casting and spurning of fibers (Chamkha, 2003).

Skidas (1961) studied the flow induced by a surface moving with a constant velocity in an ambient fluid.

Vajravelu and Hadjinicolaou (1993) and (1997) investigated a corrective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream and they showed that heat generation or absorption effect in wrong fluid is important.

Chen *et al*. (1980) did a pioneering work on heat and mass transfer. Also Gupta and Gupta (1977) studied heat and mass transfer on a stretching sheet with suction or blowing.

Muthucumaraswamy (2002) studied heat generation/absorption and magnetic effects while Chamkha (2003) studied a generation of the problem investigation by Muthucumaraswamy (2002).

Recently, Ayeni *et al*. (2004) studied an MHD flow of a uniformity stretched vertical preamble surface in the presence of heat generation/absorption which resulted from a quadratic reaction. They presented a higher order correction for the temperature field and it was shown that the correction has a maximum and that the maximum θ_{max} depends on the Prandtl number P_r and the heat absorption coefficient ϕ. In the present work, we generalized the work of Ayeni *et al*. (2004) by finding the analytical solution of the resulting system of equations of their MHD model.

Governing equations: Following the modification for an Arrhenius reaction of Ayeni *et al*. (2004), we obtain:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$V\frac{\partial u}{\partial y} = \frac{V\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{rB_0^2}{\rho}U \tag{2}$$

$$\rho CpV\frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + (C - C_\infty)^n\left\{Q_0(T - T_\infty) + Q_1(T - T_\infty)^2\right\} \tag{3}$$

$$V\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - (C - C_\infty)^n\left\{y_0(T - T\infty)^2 + y_1(T - T_\infty)^2\right\} \tag{4}$$

Where:

Y = The horizontal or transverse coordinate it is axial velocity
V = The transverse velocity
T = The fluid temperature
C = The concentration
T_∞ = The ambient temperature

C_∞ = The ambient concentration and p, $g, \beta_T, \beta_C, \vartheta, \sigma, \beta_0, D, \gamma_0, \gamma_1$

n = Density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, kinematics viscosity, fluid electrical conductivity, magnetic induction, heat generation/coefficient and the chemical reaction parameter and real number respectively

The physical boundary conditions for the problem are $U(0) = U_w$ $V(0) = -V_w$:

$$T(0) = T_w \; C(0) = C_w$$

$$Y \to \infty, U \to 0 \; T \to T_\infty \; C \to C_\infty \tag{5}$$

where, U_w, V_w>0, T_w and C_w are surface velocity, suction velocity, surface temperature and concentration respectively:

Assuming V = constant, n = 0

$$y^1 = yvw / v, \quad U^1 = \frac{u}{Uw}, \; \theta = \frac{T - T_\infty}{T_w - T_\infty}, C^1 = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

We obtain, dropping the prime:

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + G_{rT}\theta + G_{rc}C - M^2u = 0 \tag{7}$$

$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} + \varphi\theta + \varepsilon\phi_1\theta^2 = 0 \tag{8}$$

$$\frac{d^2c}{dy^2} + Sc\frac{dc}{dy} - (\delta\theta + \delta_1\theta^2) = 0 \tag{9}$$

where:

$$G_{rT} = \frac{g\beta_T V(Tw - T_\infty)}{UwV^2w}, Gr_c = g\beta_c\frac{V(C_w - C_\infty)}{UwV^2w}$$

$$\left.\begin{array}{l} Pr = \frac{\mu Cp}{K}, \quad S_c = \frac{V}{D}, \quad K = \frac{\gamma v}{V^2w} \\ M^2 = \frac{r\beta_o^2 V}{\rho V^2 w}, \quad \phi = \frac{VQ_0}{\rho Cp V^2 w} \end{array}\right\} \tag{10}$$

The dimensionless boundary conditions are:

$$U(0) = 1, \theta(0) = 1, C(0) = 1$$

$$Y \to \infty, U \to 0 \quad \theta \to 0, \quad C \to 0 \tag{11}$$

MATERIALS AND METHODS

Ayeni *et al*. (2004) solved the model for temperature only, however in the present work; we shall find solutions to the full model as in Eq. 7-9 and 11.

We seek asymptotic solution in the limit $\varepsilon \to 0$ of θ to obtain:

$$\theta = \theta_0 + \varepsilon\theta_1 + ... \tag{11}$$

Saturating Eq. 8 with 11 one obtains as in Ayeni *et al*. (2004):

$$\frac{d^2\theta_0}{dy^2} + Pr\frac{d\theta_0}{dy} + \varphi\theta_0 = 0 \tag{12}$$

$$\theta_0(0) = 1 \qquad \theta_0(\infty) = 0 \tag{13}$$

$$\frac{d^2\theta_1}{dy^2} + Pr\frac{d\theta_1}{dy} + \phi\theta_1 + \phi_1\theta_0^2 = 0 \tag{14}$$

$$\theta_1(0) = \theta_1(\infty) = 0 \tag{15}$$

The solutions of (12-15) are:

$$\theta_0 = \exp(-my) \tag{16}$$

$$\theta_1 = \frac{\phi_1}{4m^2 - 2Pr\,m + \phi}\left[\exp(-my) - \exp(-2my)\right] \tag{17}$$

Where:

$$m = \tfrac{1}{2}\left[Pr + \sqrt{Pr^2 - 4\phi}\right] \tag{18}$$

hence the solution of (9) is given as:

$$\theta = \theta_0 + \varepsilon\phi_1$$

$$\text{i.e., } \theta = \exp(-my) + \frac{\varepsilon\phi_1}{4m^2 - 2Pr\,m + \phi}\begin{bmatrix}\exp(-my) - \\ \exp(-2my)\end{bmatrix} \tag{19}$$

Using (19 and 9) has a solution of the form:

$$C = \alpha_2 e^{-S_c y} + \alpha_3 e^{-my} + \alpha_4 e^{-2my} + \alpha_5 e^{-3my} + \alpha_6 e^{-4my}$$

Where:

$$\alpha_3 = \frac{\delta_0 A_1}{m(m - S_c)}$$

$$\alpha_4 = \frac{\delta_1 A_1^2 - \delta_0 A_2}{2m(2m - S_c)}$$

$$\alpha_5 = \frac{-2\delta_1 A_1 A_2}{3m(3m - S_c)}$$

$$\alpha_6 = \frac{\delta_1 A_2^2}{4m(4m - S_c)}$$

$$A_1 = \left[1 + \frac{\varepsilon\phi_1}{4m^2 - 2m\Pr + \phi}\right]$$

$$A_2 = \frac{\varepsilon\phi_1}{4m^2 - 2\Pr m + \phi}$$

Also the solution of (7) is given by:

$$U = \{1 - (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)\} e^{r_2 y} + \lambda_1 e^{-S_c y} + \lambda_2 e^{-my} + \lambda_3 e^{-2my} + \lambda_4 e^{-3my} + \lambda_5 e^{-4my}$$

Where:

$$r_2 = -1 - \frac{\sqrt{1 + 4M^2}}{2}$$

$$\lambda_1 = \frac{-G_{rc}}{S_c^2 - S_c - M^2}$$

$$\lambda_2 = -\frac{(G_{rT} A_1 + G_{rc}\alpha_3)}{m^2 - m - M^2}$$

$$\lambda_3 = \frac{(G_{rT} A_2 - G_{rc}\alpha_4)}{4m^2 - 2m - M^2}$$

$$\lambda_4 = \frac{-G_{rc}\alpha_5}{9m^2 - 3m - M^2}$$

$$\lambda_5 = \frac{-G_{rc}\alpha_6}{16m^2 - 4m - M^2}$$

RESULTS AND DISCUSSION

The results in Table 1 were obtained using Eq. 19.

Figure 1 presents axial velocity profiles for various combinations of the parameters K, Pr, Sc and ϕ in the absence of a magnetic filed (M = 0) and in the presence of both thermal and concentration buoyancy effects.

It should be noted that K>0 indicates a destructive chemical reaction while K<0 corresponds to a generative chemical reaction. Also, ϕ<0 indicates heat generation while ϕ>0 corresponds to heat absorption. Similarly, K = 0 and ϕ = 0 indicate no chemical reaction and no heat generation/absorption effects respectively. It is easily seen in Fig. 1 that for a destructive chemical reaction (K = 2.0) with Sc = 2.0, increasing Pr produces lower fluid velocities.

Also, for lower values of Pr (Pr = 0.71) regardless of the value of K, a destructive peak in the velocity profile is predicted. However, this does not occur for relatively higher values of Pr (Pr = 7.0).

The presence of the peak indicates that the maximum value of velocity occurs in the body of fluid close to the surface and not at the surface. Also from these increases in the values of Sc results in reduced flow velocities.

Table 1: Velocity profiles for various combinations of the parameters K, Pr, Sc and ϕ

	ϕ	K	Sc	Pr
1.	-0.1	2.0	0.6	0.71
2.	-0.1	-0.2	2.0	0.71
3.	-0.1	0.0	2.0	0.71
4.	-0.1	2.0	2.0	0.71
5.	-0.1	2.0	0.6	7.00
6.	-0.1	2.0	1.0	7.00
7.	-0.1	2.0	2.0	7.00
00A	0.0	2.0	0.6	0.71
07A	0.0	2.0	0.6	7.00
1E	1.0	2.0	0.6	7.00
1G	1.0	2.0	2.0	7.00

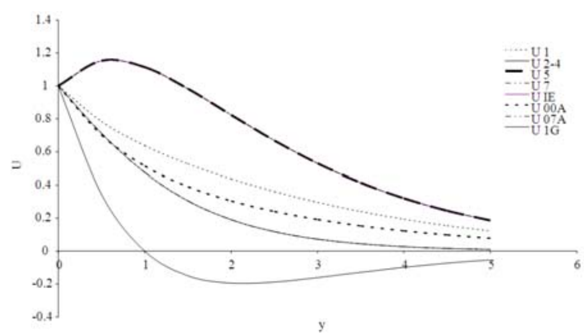

Fig. 1: Effects of various parameters on the velocity profiles

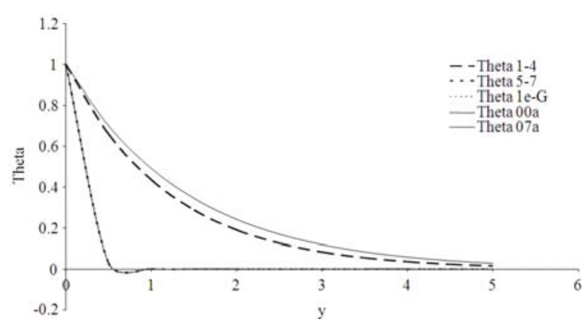

Fig. 2: Effects of Pr and ϕ on the temperature profiles

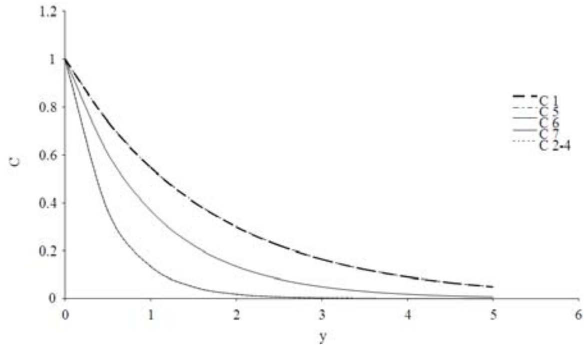

Fig. 3: Effects of K and Sc on the concentration profiles

Furthermore, it is easily seen that increases in the values of K cause reductions in the fluid velocities. Similarly, increases in the value of ϕ cause the fluid to move at a slower rate. Also the effect of heat generation ($\phi = 0$) on the velocity profile is more pronounced than that of heat absorption.

In Fig. 2, the effect of both Pr and ϕ on the temperature profiles are illustrated it is easily dissemble that as either of Pr or ϕ increases, the fluid temperature decreases. Also, the effect of increasing or decreasing ϕ for Pr = 0.71 is much more pronounced than that corresponding to Pr = 7.0.

Figure 3 exhibits the effect of both K and Sc on the species concentration profile. As expected the presence of a chemical reaction significantly affects the concentration profile. Indeed, as K increases, considerable reduction in the concentration is predicted. Also, increasing in the value of Sc produces lower concentration values.

Values for variables used in the graph:

$$\varepsilon = 0.01, \quad \delta = 0.001, \quad \delta_1 = 0.005, \quad \phi_1 = 0.01, \quad M = 0,$$
$$Gr_t = 1.0, \quad Gr_c = 1.0$$

CONCLUSION

Analytical solutions for an MHD flow of a uniformly stretched vertical preamble surface in the presence of heat generating absorption which resulted from a quadratic reaction were reported, from the graphical results, the following conclusions were deduced:

- The fluid velocity decreased as either of the Prandtl number, the Schmidt number or the strength of the magnitude field was increased and increased as either of the thermal or concentration buoyancy effects were increased
- The fluid velocity increased during a generative chemical reaction and decreased during a destructive one. Also, the presence of heat generation effects increased the fluid velocity while the presence of heat absorption effects decreased it

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