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The Conditional Sequence Information Function

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Abstract: Problem statement: A great deal of attention has been given to the theory of information. It has found its applications in science especially in the area of Biothecnology. Previous studies on the subject has been limited to either conditional or sequence problem solving. This study combines both conditional and sequence properties of the information function. By doing this, researchers can find solutions to more problems that are applicable in real life. **Approach:** First, properties of the dynamical systems and the information function defined by Shannon has been provided, Then, the conditional sequence information function of a dynamical system together with its proof was presented. **Result:** This new function now exists now and ready for the use in many real life problems such as finding solutions to DNA sequence with conditional sickness. **Conclusion:** This new function created opens a new avenue to researches in solving more complex problems by using its developed properties.

Key words: Entropy, dynamical systems, information function, conditional sequence information function

INTRODUCTION

Currently many researchers are investigating bioinformation using very complex mathematical functions. Of particular interest to us is the information function defined by Shannon (1948). Both Khinchin (1957) and Shannon (1948) have investigated the properties of this information function. Brown (1976) and Gray and Davidson (1970) defined the entropy function of dynamical systems and investigated its properties. Tok (1986) defined the fuzzy information function and investigated its properties. Moreover, Newton (1970a; 1970b); Walters (1975; 2000) and Guzide (1990) (in Turkish) defined conditional sequence entropy and sequence entropy functions and investigated their properties.

In this study, we give the definition of a conditional sequence information function and prove that it exists.

First we give the some properties of dynamical systems necessary to our discussion and states the sequence information function and list its properties. The conditional sequence entropy defined by Zhang (1993). We then define the conditional sequence information function and finish with a proof of its existence.

Dynamical systems and information function: We will give very important background support and definition of dynamical system and information Function.

Definition 1: A measure-preserving dynamical system is defined as a probability space and a measure-preserving transformation on it. In more detail, it is a system (X, A, μ,T) with the following structure:

- X is a set
- A is aσ-algebra over X
- μ (A)→[0,1] is a probability measure, so that μ (X) = 1
- T:X→Y is a measurable transformation which preserves the measure μ

Definition 2: Consider two dynamical systems (X, A, μ, T) and (Y, B, υ, S). Then a mapping ϕ: X→Y is a homomorphism of dynamical systems if it satisfies the following three properties:

- The map φ is measurable
- For each B∈ B, one has $\mu(\varphi^{-1}B) = \upsilon(B)$
- For μ-almost all x∈ X, one has φ (Tx) = S(φx)

The system (Y, B, υ, S) is then called a factor of (X, A, μ, T).

The map φ is an isomorphism of dynamical systems if, in addition, there exists another mapping ψ: Y→X that is also a homomorphism, which satisfies:

- For μ-almost all x∈ X, one has x = ψ (φx)
- For υ-almost all y∈ Y, one has y = φ (ψy)

Definition 3:

- Consider a dynamical system (X, A, μ,T) and P is a σ-measure on X. I(P) = -logμ(A) is called information of P where $A\in P$
- I(P,x) = $\sum_{A\in P} \chi_A \log \mu(A)$ is called information function where $\forall x \in X$

Example 1: Consider dynamical system (X, A, μ, T). Then If ψ: X$\to$X: is the identity, then I(ψ) = 0. This is because $I(\psi, A) = \lim(1/n)\mu(A)$ = 0. Also $T^p = \psi$ for some $p \neq 0$ then I(T) = 0.

Now we are ready to give conditional information function definition.

Definition 4: Consider a dynamical system (X, A, μ,T). $P = \{A_1, A_2, ..., A_n\}$ and $Q = \{B_1, B_2, ..., B_n\}$ be two X. Then $\mu\in$(X, A, μ,T) and $\forall j$ where $\mu(B_j) \geq 0$ then function:

$$I(P|Q) = I_\mu(P|Q) = -\sum_{j=1}^m \mu(B_j) \sum_{i=1}^n \frac{\mu(A_i \cap B_j)}{\mu(B_j)} \log \frac{\mu(A_i \cap B_j)}{\mu(B_j)}$$
$$= \sum_{j=1}^m (B_j) \sum_{i=1}^n \eta(\mu(A_i | B_j))$$
$$= -\sum_{i=1}^n \sum_{j=1}^m \mu(B_j)\mu\,(A_i \cap B_j) \log \mu(B_j)\mu(A_i | B_j)$$

called conditional information function of P based on Q.

Now we give supporting theorem, lemma and definition to main theorem.

Lemma 1: If $(a_n)_{n\geq 1}$ and $a_n \geq 0$ and $a_{(a+m)} \leq a_n + a_m$ for $\forall n, m$, then $\lim_{n\to\infty} \frac{1}{n} a_n$ exits and limit $(a_n)_{n\geq 1}$ equals to inf $\frac{1}{n} a_n$.

Theorem 1: Consider a dynamical system (X, A, μ,T), T be an invariant transformation and $P\in Z$. $\lim_{n\to\infty}$ sup $\frac{1}{n} I(V_{i=1}^n T^{t_i} P, x)$ exists where $\Gamma = \{t_i\}_1^\infty$ and $\forall x \in X$, as proven in (Guzide, 1990).

Now we define the sequence information function.

Definition 5: Consider a dynamical system (X, A, μ,T)and $P\in Z$ where $\forall x \in X$. $I_\Gamma(T, P, x)$ is called the sequence information function of P under T and exits based on Theorem 1.

Conditional sequence information function: Consider $\Gamma = \{t_n\}_{n=1}^\infty$ is sequence integer numbers where n$\geq$0 and $t_1 = 0$.

Theorem 2: Consider a dynamical system (X, A, μ, T), (Y, B, υ, T′) is a factor of (X, A, μ,T), B is a σ-algebra invariant under T of A and $\Gamma = \{t_n\}_{n-1}^\infty$ be integer. Then $P\in Z$ and $\lim_{n\to+\infty} \sup \frac{1}{n} I(V_{i=1}^n T^{t_i} P | B)$ exits.

Proof 1: Consider $a_n = I(V_{i=1}^n T^{t_i} P | B)$ and then need to show $(a_n)_{n\geq 1}$, $a_n \geq 0$, and $a_{n+m} \leq a_n + a_m$. For $P\in Z$ and from definition 4:

$$I(V_{i=1}^n T^{t_i} P | B) = -\sum_{B\in B} \mu(B) \Sigma_{A\in V_{i=1}^n T^{t_i} P} \frac{\mu(A\cap B)}{\mu(B)} \log \frac{\mu(A\cap B)}{\mu(B)}$$
$$= \sum_{B\in B} \mu(B) \Sigma_{A\in V_{i=1}^n T^{t_i} P} \eta(\mu(A|B))$$

Obvious $a_n \geq 0$. Need to investigate σ-addition of $(a_n)_{n\geq 1}$. Let $t_1 = 0$ and $\{t_i\}_i$ be sequence integer number where $n+1 \leq i \leq n+m$. From proof of sequence and conditional information function and using same proof of sequence information function Lemma 1 in (Guzide, 1990). First investigation is:

$$\begin{aligned}
a_{n+m} &= I(V_{i=1}^{n+m} T^{t_i} P | B) \\
&= \int I_y(V_{i=1}^{n+m} T^{t_i} P) d_\gamma \\
&= \int I_y(V_{i=1}^n T^{t_i} P \ V \ V_{i=n+1}^{n+m} T^{t_i} P) d_\gamma \\
&\leq \int [I_y(V_{i=1}^n T^{t_i} P) + I_y(V_{i=n+1}^{n+m} T^{t_i} P)] d_\gamma \\
&= \int I_y(V_{i=1}^n T^{t_i} P) d_\gamma + \int I_y(V_{i=n+1}^{n+m} T^{t_i} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(T^{-t_j}(V_{i=n+1}^{n+m} T^{t_i} P)) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(V_{i=n+1}^{n+m} T^{t_i - t_j} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(V_{k=1}^{j-n-1} T^{(t_{n+k} - t_j)} \\
&\quad PVT^{(t_i - t_j)} P \ V \ V_{\ell=j-n+1}^m T^{(t_{n+\ell} - t_j)} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(V_{k=1}^{j-n-1} T^{(t_{n+k} - t_j)} P \\
&\quad VT^{(t_i - t_j)} PV \ V_{\ell=j-n+1}^m T^{t_\ell} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(T^{t_1} PVT^{t_2} PV.... \\
&\quad VT^{t_{j-n}} PVT^{t_{j-n+1}} PV...VT^{t_m} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + \int I_y(V_{i=1}^m T^{t_i} P) d_\gamma \\
&= I(V_{i=1}^n T^{t_i} P | B) + I(V_{i=1}^m T^{t_i} P | B) \\
&= a_n + a_m
\end{aligned}$$

Moreover sequence $(a_n)_{n\geq 1}$ satisfies all conditions. Therefore:

$$\lim_{n\to+\infty}\sup\frac{1}{n}I(V_{i=1}^{n}T^{t_i}P\,|\,B)$$

exits and:

$$I_\Gamma(T,P\,|\,B) = \lim_{n\to+\infty}\ \sup\frac{1}{n}I(V_{i=1}^{n}T^{t_i}P\,|\,B)$$

Definition 6: The function $I_\Gamma(T,P\,|\,B)$ called conditional information function of P based on T.

MATERIALS AND METHODS

This problem only use previous functions related dynamical systems. So far only certain level problems are solved by previous functions. Now researchers more advanced and complex functions to solve complex problems.

RESULTS AND DISCUSSION

The conditional information function can be used for real life problem with conditional and sequence information together. For future research, the investigations will be properties of new conditional sequence function.

With new function the researcher will invastigate both conditional and sequence case of problems. Next step one research and proof the properties of The Conditional Sequence Information Function.

CONCLUSION

New function will be provided real life problem to solve more complex problem. Also this function has properties we did not investigate yet. Researcher will use this function and properties for complex real life problems.

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