

Stochastic Analysis of a Compound Redundant System Involving Human Failure

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Abstract: This study deals with a compound standby redundant system consisting of three sub-systems A, B, and C connected in series. The sub-system B consists of one main unit and the other is its standby redundant unit. These units further consist of two sub-units connected in series. The sub-units of main unit are connected to sub-units of standby unit through imperfect switching over device. Failure of all sub-systems and repair rate of switching over devices are exponential while repairs of all sub-systems are distributed quite generally. The various reliability parameters have been computed and analyzed by tabular and graphical illustrations.

Key words: M.T.T.F., availability, reliability, profit function, standby redundancy

INTRODUCTION

Reliability is an important concept at the planning, design and operation stages of various complex systems. As long as man has built things, he has wanted to make them as reliable as possible. In practice, we come across with a number of complex systems consisting of one or more parts, failure of any of the parts results in the reduction of efficiency of whole systems or the complete failure of the system and as a result of it, the reliability of the system reduces. The better maintenance of such parts originate better reliability and then only we can achieve the markets demands of reliability, functionality, price and performance of that system. On the other hand it may not be economical to obtain higher order of reliability always through any amount of maintenance. Thus introducing redundant parts and providing maintenance and repair at the time of need may achieve high degree of reliability. In a redundant system, some additional paths are created for the proper functioning of the system. If all the redundant parts start working together at the time of operation, then it is termed as parallel redundancy. A standby redundant system is the one in which one operating unit is followed by spare units called standbys. On the failure of the operating unit, a standby unit is switched on by perfect or imperfect switching device. In the present discussion, the authors have considered a compound system consisting of three sub-systems A, B and C. The sub-system B consists of two units, one is main and the other is its standby. These units further consist of two sub-units viz, (B11, B21) in main unit and (B12, B22) in its standby redundant system. The sub-system B also has two imperfect switching devices S1 and S2. S1 connects B11 and B22 and S2 connects B21 and B12. B12 and

B22 sub-units are same as B11 and B21 respectively. Initially, in sub-system B, the sub-units B11 and B21 are assumed to be in operation. If B11 fails then B12 and B21 may begin to operate through a switching device S2. If B21 fails then B11 and B22 may begin to operate through a switching device S1. The failure in sub-system C requires waiting time for repair. The system will be in down state due to failure of sub-system A or C or occurrence of any human error. Also, due to failure of two sub-units of one unit and one sub-unit of other unit of sub-system B, the system suffers complete break down. The Laplace transforms of the time dependent probabilities of the system being in various states have been obtained by employing the supplementary variable technique. Various reliability parameters have been computed and some tabular and graphical illustrations are also given at the end of the paper. The state transition diagram of the system is shown in Fig. a.

Assumptions

- * Initially, the system works perfectly.
- * The system consists of three subsystems A, B and C connected in series.
- * The Sub-system B consists of two units, one main unit and other in standby mode.
- * The main unit of subsystem B further consists of two sub-units B11 and B21 connected in series. Similarly, standby unit consists of two sub-units B12 & B22 connected in series.
- * The Sub-unit B11 is connected to sub-unit B22 by switch S1 and sub-unit B21 is connected to sub-unit B12 by switch S2.
- * Imperfect switching device is assumed.
- * All the repairs of sub-systems are distributed quite generally while the repair rate of switching over

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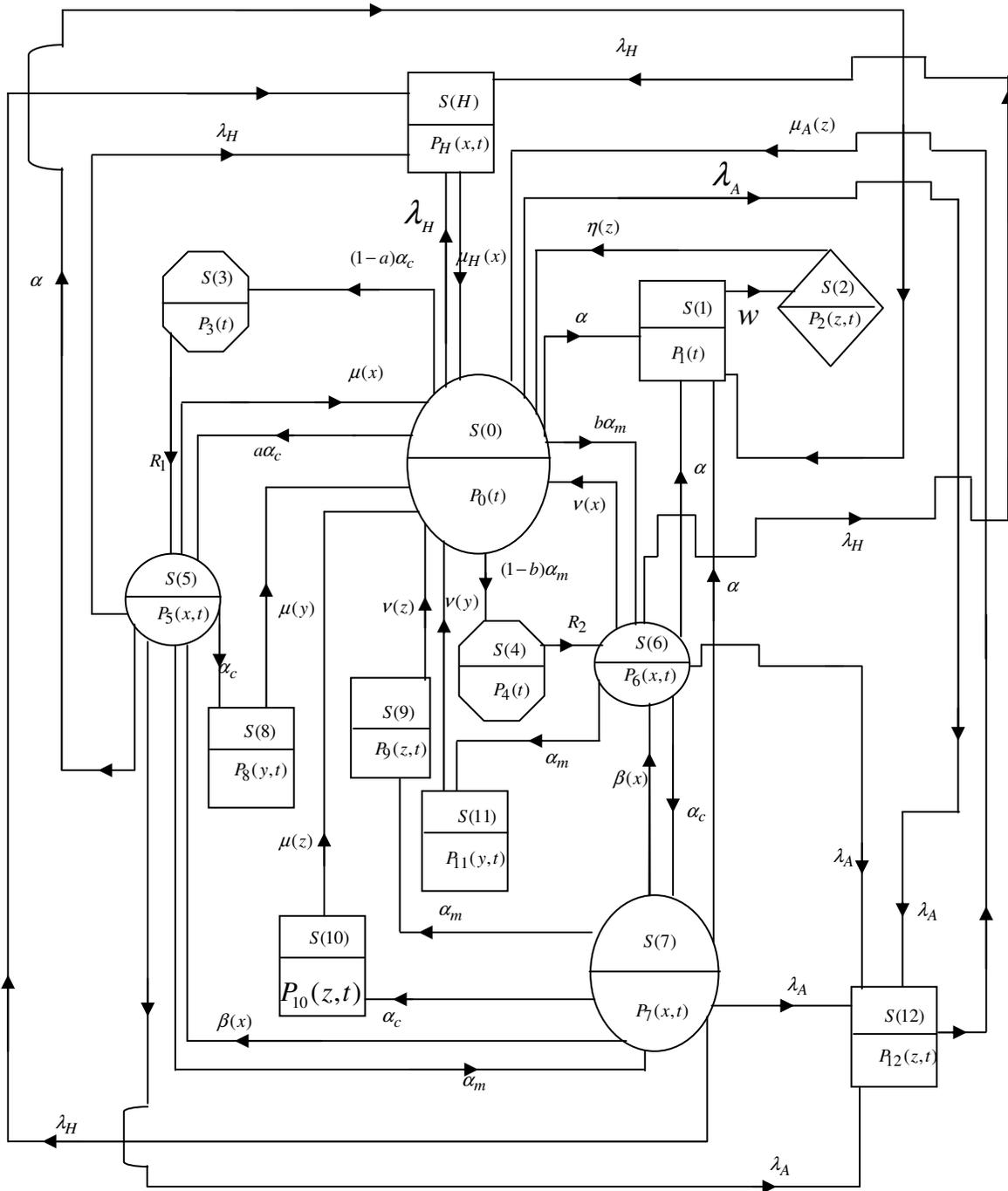


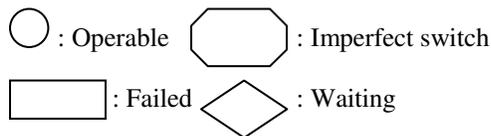
Fig. a: State transition diagram

- devices and the failures of sub-systems are exponentially distributed.
- * All the units recover their functioning perfectly after repair.
- * All the repairs are perfect i.e., the repair facility never does any damage to the units.
- * A failed subsystem is repaired at a single service channel.
- * At time $t=0$, B11 and B21 are operating and B12 and B22 are in standby mode.
- * As soon as the operating unit fails, it is replaced by its standby unit.
- * The system ceases to function due to failure of subsystem A or C or due to occurrence of any human error.

- * The system suffers complete break down due to failure of sub-systems B21, B22 or B11, B12 or any three sub-units of sub-system B at a single state.
- * The sub-system C requires waiting time for repair.
- * In state S(7) of the system no priority is given to any sub-unit i.e., B11, B21 and repair rate of these sub-units is assumed to be identical.
- * Standby sub-units in sub-system B are assumed to be perfect as long as they are in standby mode.

Notations

- α : Constant failure rate of subsystem C.
- $\eta(z)$: Repair rate of sub-system C when the system is in the state S(2).
- w : Constant waiting time to repair subsystem C.
- α_c : Failure rate of sub-unit B21 or B22 of sub-system B.
- α_m : Failure rate of sub-unit B11 or B12 of sub-system B.
- $\lambda_H, \mu_H(x)$: Failure and repair rate of the system due to human failure.
- $\lambda_A, \mu_A(z)$: Failure and repair rate of sub-system A.
- $\mu(x)$: Repair rate of sub-unit B21 when the system is in state S(5).
- $\mu(y)$: Repair rate of sub-unit B21 and B22 when the system is in state S(8).
- $\mu(z)$: Repair rate of sub-unit B11, B21 and B22 when the system is in state S(10).
- $\beta(x)$: Repair rate of sub-unit B11 or B21 when the system is in state S(7).
- $\nu(x)$: Repair rate of sub-unit B11 when the system is in state S(6).
- $\nu(y)$: Repair rate of sub-unit B11 and B12 when the system is in state S(11).
- $\nu(z)$: Repair rate of sub-unit B11, B21 and B12 when the system is in state S(9)
- x, y, z : Elapsed repair time.
- a, b : Probability of successful operation of switching over devices S1 and S2 respectively.
- R_1, R_2 : Constant repair rate of switching over devices S1 and S2 respectively.



State probability description

$P_0(t)$: Probability that the system is in operable state S(0) at time 't'.

- $P_1(t)$: Probability that the system is in failed state S(1) at time t.
- $P_2(z,t)\Delta$: Probability that the system is in waiting state S(2) with elapsed repair time lying in the interval $(z, z + \Delta)$.
- $P_3(t)$: Probability that the system is in failed state S(3) at any time 't'
- $P_4(t)$: Probability that the system is in failed state S(4) at any time 't'.
- $P_5(x,t)\Delta$: Probability that the system is in operable state S(5) with elapsed repair time lying in the interval $(x, x + \Delta)$.
- $P_6(x,t)\Delta$: Probability that the system is in operable state S(6) with elapsed repair time lying in the interval $(x, x + \Delta)$.
- $P_7(x,t)\Delta$: Probability that the system is in operable state S(7) with elapsed repair time lying in the interval $(x, x + \Delta)$.
- $P_8(y,t)\Delta$: Probability that the system is in failed state S(8) with elapsed repair time lying in the interval $(y, y + \Delta)$.
- $P_9(z,t)\Delta$: Probability that the system is in failed state S(9) with elapsed repair time lying in the interval $(z, z + \Delta)$.
- $P_{10}(z,t)\Delta$: Probability that the system is in failed state S(10) with elapsed repair time lying in the interval $(z, z + \Delta)$.
- $P_{11}(y,t)\Delta$: Probability that the system is in failed state S(11) with elapsed repair time lying in the interval $(y, y + \Delta)$.
- $P_{12}(z,t)\Delta$: Probability that the system is in failed state S(12) with elapsed repair time lying in the interval $(z, z + \Delta)$.
- $P_H(x,t)\Delta$: Probability that the system is in failed state S(H) with elapsed repair time lying in the interval $(x, x + \Delta)$.

Formulation of mathematical model: By elementary probability consideration and continuity arguments, the following difference-differential equations governing the behaviour of the system may be hold good:

$$\left[\frac{d}{dt} + a\alpha_c + b\alpha_m + (1-b)\alpha_m + \lambda_A + \alpha + \lambda_H + (1-a)\alpha_c \right] P_0(t) = \int_0^\infty \mu(x).P_5(x,t)dx + \int_0^\infty \nu(x).P_6(x,t)dx + \int_0^\infty \nu(y).P_{11}(y,t)dy + \int_0^\infty \mu_A(z).P_{12}(z,t)dz + \int_0^\infty \mu_H(z).P_H(x,t)dx + \int_0^\infty \eta(z).P_2(z,t)dz + \int_0^\infty \mu(z).P_{10}(z,t)dz + \int_0^\infty \nu(z).P_9(z,t)dz + \int_0^\infty \mu(y).P_8(y,t)dy \tag{1}$$

$$\left[\frac{d}{dt} + w \right] .P_1(t) = \alpha .P_0(t) + \alpha \int_0^\infty P_5(x,t) dx + \alpha \int_0^\infty P_6(x,t) dx + \alpha \int_0^\infty P_7(x,t) dx \tag{2}$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \eta(z) \right] .P_2(z,t) = 0 \tag{3}$$

$$\left[\frac{d}{dt} + R_1 \right] .P_3(t) = (1-a)\alpha_c .P_0(t) \tag{4}$$

$$\left[\frac{d}{dt} + R_2 \right] .P_4(t) = (1-b)\alpha_m .P_0(t) \tag{5}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \mu(x) \right] .P_5(x,t) = 0 \tag{6}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \nu(x) \right] .P_6(x,t) = 0 \tag{7}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + 2\beta(x) \right] .P_7(x,t) = 0 \tag{8}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \mu(y) \right] .P_8(y,t) = 0 \tag{9}$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \nu(z) \right] .P_9(z,t) = 0 \tag{10}$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \mu(z) \right] .P_{10}(z,t) = 0 \tag{11}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \nu(y) \right] .P_{11}(y,t) = 0 \tag{12}$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \mu_A(z) \right] .P_{12}(z,t) = 0 \tag{13}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_H(x) \right] .P_H(x,t) = 0 \tag{14}$$

Boundary conditions

$$P_2(0,t) = w .P_1(t) \tag{15}$$

$$P_5(0,t) = R_1 .P_3(t) + a .\alpha_c .P_0(t) + \int_0^\infty \beta(x) P_7(x,t) dx \tag{16}$$

$$P_6(0,t) = b .\alpha_m .P_0(t) + R_2 .P_4(t) + \int_0^\infty \beta(x) P_7(x,t) dx \tag{17}$$

$$P_7(0,t) = \alpha_c \int_0^\infty P_6(x,t) dx + \alpha_m \int_0^\infty P_5(x,t) dx \tag{18}$$

$$P_8(0,t) = \alpha_c \int_0^\infty P_5(x,t) dx \tag{19}$$

$$P_9(0,t) = \alpha_m \int_0^\infty P_7(x,t) dx \tag{20}$$

$$P_{10}(0,t) = \alpha_c \int_0^\infty P_7(x,t) dx \tag{21}$$

$$P_{11}(0,t) = \alpha_m \int_0^\infty P_6(x,t) dx \tag{22}$$

$$P_{12}(0,t) = \lambda_A \int_0^\infty P_6(x,t) dx + \lambda_A .P_0(t) + \lambda_A \int_0^\infty P_5(x,t) dx + \lambda_A \int_0^\infty P_7(x,t) dx \tag{23}$$

$$P_H(0,t) = \lambda_H .P_0(t) + \lambda_H \int_0^\infty P_5(x,t) dx + \lambda_H \int_0^\infty P_6(x,t) dx + \lambda_H \int_0^\infty P_7(x,t) dx \tag{24}$$

Initial conditions: $P_0(0) = 1$ and other state probabilities are zero.

Solution of the model: Taking Laplace transform of (1) to (24) and on further simplification one may obtain:

$$\overline{P_0}(s) = \frac{1}{A(s)} \tag{25}$$

$$\overline{P_1}(s) = K_5(s) \frac{1}{A(s)} \tag{26}$$

$$\overline{P_2}(s) = K_6(s) \frac{1}{A(s)} \tag{27}$$

$$\overline{P_3}(s) = K_7(s) \frac{1}{A(s)} \tag{28}$$

$$\overline{P_4}(s) = K_8(s) \frac{1}{A(s)} \tag{29}$$

$$\overline{P_5}(s) = K_{12}(s) \frac{1}{A(s)} \tag{30}$$

$$\overline{P_6}(s) = K_{13}(s) \frac{1}{A(s)} \tag{31}$$

$$\overline{P_7}(s) = K_{14}(s) \frac{1}{A(s)} \tag{32}$$

$$\overline{P_8}(s) = K_{15}(s) \frac{1}{A(s)} \tag{33}$$

$$\overline{P_9}(s) = K_{16}(s) \frac{1}{A(s)} \tag{34}$$

$$\overline{P_{10}}(s) = K_{17}(s) \frac{1}{A(s)} \tag{35}$$

$$\overline{P_{11}}(s) = K_{18}(s) \cdot \frac{1}{A(s)} \tag{36}$$

$$\overline{P_{12}}(s) = K_{20}(s) \cdot \frac{1}{A(s)} \tag{37}$$

$$\overline{P_H}(s) = K_{21}(s) \cdot \frac{1}{A(s)}, \tag{38}$$

Operational availability and non-availability: The Laplace transform of the probabilities that the system is in operable and down state at time 't' can be evaluated as follows:

$$\overline{P_{up}}(s) = \frac{1}{A(s)} \{1 + K_{12}(s) + K_{13}(s) + K_{14}(s)\} \tag{39}$$

$$\overline{P_{down}}(s) = \frac{1}{A(s)} \left\{ \begin{aligned} &K_5(s) + K_6(s) + K_7(s) + K_8(s) + K_{15}(s) + K_{16}(s) + K_{17}(s) \\ &+ K_{18}(s) + K_{20}(s) + K_{21}(s) \end{aligned} \right\} \tag{40}$$

It is worth noticing that $\overline{P_{up}}(s) + \overline{P_{down}}(s) = \frac{1}{s}$.

Where,

$$A(s) = (s + \alpha_c + \alpha_m + \lambda_A + \lambda_H + \alpha) -$$

$$K_{22}(s) - K_{23}(s) - K_{24}(s) -$$

$$K_{25}(s) - K_{26}(s) - K_{27}(s)$$

$$- K_{28}(s) - K_{29}(s) - K_{30}(s)$$

$$K_1(s) = \frac{1}{(s + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \mu(x))}$$

$$K_2(s) = \frac{1}{(s + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \nu(x))}$$

$$K_3(s) = \frac{1}{(s + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + 2\beta(x))}$$

$$K_4(s) = \beta(x) \cdot K_3(s)$$

$$K_5(s) = \frac{\alpha}{s + w} + \alpha \cdot K_{12}(s) + \alpha \cdot K_{13}(s) + \alpha \cdot K_{14}(s)$$

$$K_6(s) = \frac{w}{(s + \eta(z))} \cdot K_5(s)$$

$$K_7(s) = \frac{(1-a)\alpha_c}{(s + R_1)}$$

$$K_8(s) = \frac{(1-b)\alpha_m}{(s + R_2)}$$

$$K_9(s) = 1 - \alpha_c \cdot K_4(s) - \alpha_m \cdot K_4(s) - K_1(s)$$

$$K_{10}(s) = K_2(s) \{ \alpha_c \cdot b \cdot \alpha_m + \alpha_c \cdot R_2 \cdot K_8(s) \}$$

$$+ K_1(s) \{ \alpha_m \cdot R_1 \cdot K_7(s) + \alpha_m \cdot a \cdot \alpha_c \}$$

$$K_{11}(s) = \frac{K_{10}(s)}{K_9(s)}$$

$$K_{12}(s) = K_1(s) \{ K_7(s) \cdot R_1 + a \cdot \alpha_c + K_{11}(s) \cdot K_4(s) \}$$

$$K_{13}(s) = K_2(s) \{ K_8(s) \cdot R_2 + b \cdot \alpha_m + K_{11}(s) \cdot K_4(s) \}$$

$$K_{14}(s) = K_{11}(s) \cdot K_3(s)$$

$$K_{15}(s) = \frac{\alpha_c \cdot K_1(s)}{(s + \mu(y))} \{ K_7(s) \cdot R_1 + a \cdot \alpha_c + K_{11}(s) \cdot K_4(s) \}$$

$$K_{16}(s) = \frac{\alpha_m \cdot K_{11}(s) \cdot K_3(s)}{(s + \nu(z))}$$

$$K_{17}(s) = \frac{\alpha_c \cdot K_{11}(s) \cdot K_3(s)}{(s + \mu(z))}$$

$$K_{18}(s) = \frac{\alpha_m \cdot K_{13}(s)}{(s + \nu(y))}$$

$$K_{19}(s) = 1 + K_{12}(s) + K_{13}(s) + K_{14}(s)$$

$$K_{20}(s) = \frac{\lambda_A \cdot K_{19}(s)}{(s + \mu_A(z))}$$

$$K_{21}(s) = \frac{\lambda_H \cdot K_{19}(s)}{(s + \mu_H(x))}$$

$$K_{22}(s) = \frac{\mu(x)}{(s + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \mu(x))}$$

$$\{ K_7(s) \cdot R_1 + a \cdot \alpha_c + K_{11}(s) \cdot K_4(s) \}$$

$$K_{23}(s) = \frac{\nu(x)}{(s + \alpha_c + \lambda_H + \alpha_m + \lambda_A + \alpha + \nu(x))}$$

$$\{ K_8(s) \cdot R_2 + b \cdot \alpha_m + K_{11}(s) \cdot K_4(s) \}$$

$$K_{24}(s) = \alpha_m \cdot K_{13}(s) \cdot \frac{\nu(y)}{(s + \nu(y))}$$

$$K_{25}(s) = \lambda_A \cdot K_{19}(s) \cdot \frac{\mu_A(z)}{(s + \mu_A(z))}$$

$$K_{26}(s) = \lambda_H \cdot K_{19}(s) \cdot \frac{\mu_H(x)}{(s + \mu_H(x))}$$

$$K_{27}(s) = \frac{w \cdot \alpha \cdot \eta(z)}{(s + w)(s + \eta(z))}$$

$$K_{28}(s) = \alpha_c \cdot K_{11}(s) \cdot K_3(s) \cdot \frac{\mu(z)}{(s + \mu(z))}$$

$$K_{29}(s) = \alpha_m \cdot K_{11}(s) \cdot K_3(s) \cdot \frac{\nu(z)}{(s + \nu(z))}$$

$$K_{30}(s) = \frac{\alpha_c \cdot K_1(s) \cdot \mu(y)}{(s + \mu(y))}$$

$$\{ K_7(s) \cdot R_1 + a \cdot \alpha_c + K_{11}(s) \cdot K_4(s) \}$$

Ergodic behaviour: Using Abel's lemma in Laplace transform, viz $\lim_{s \rightarrow 0} (s \cdot \bar{f}(s)) = \lim_{t \rightarrow \infty} f(t) = f(\text{say})$,

provided that the limit on the right hand side exists, the time independent up and down state probabilities are obtained as follows:

$$P_{up} = \frac{1}{A(0)} \{ K_{12}(0) + K_{13}(0) + K_{14}(0) \} \tag{41}$$

$$P_{down} = 1 - P_{up} \tag{42}$$

Where

$$A'(0)=1-K'_{22}(0)-K'_{23}(0)-K'_{24}(0)-K'_{25}(0) \\ -K'_{26}(0)-K'_{27}(0)-K'_{28}(0)-K'_{29}(0)-K'_{30}(0)$$

Reliability: The reliability of the system is given as follows:

$$R(t) = e^{-a_1 t} + (a\alpha_c + b\alpha_m).t.e^{-a_1 t} \\ + (\alpha_c.b\alpha_m + \alpha_m.\alpha_c.a).\frac{t^2}{2}.e^{-a_1 t} \quad (43)$$

Where $a_1 = \alpha_c + \alpha_m + \lambda_A + \lambda_H + \alpha$

M.T.T.F: The Mean Time To Failure of the system is given by

$$M.T.T.F. = \int_0^{\infty} R(t)dt = \frac{1}{a_1} + \frac{(a\alpha_c + b\alpha_m)}{a_1^2} \\ + \frac{(a\alpha_c.\alpha_m + b\alpha_m.\alpha_c)}{a_1^3} \quad (44)$$

Profit function

i. For Non – repairable system, the Profit Function $H(t)$ in the interval (0,t) is given as follows:

$$H(t) = C_1 \cdot \int_0^t R(t)dt - C_2.t - C_3 \quad (45)$$

ii. For repairable system, the profit function $H(t)$ in the interval (0,t) is given by

$$H(t) = C_1 \cdot \int_0^t P_{up}(t)dt - C_2.t - C_3 \quad (46)$$

where C_1, C_2 and C_3 are revenue per unit time, service cost per unit time and system establishment cost respectively.

Numerical computation

1. Availability analysis: Setting the values

$$\alpha_c = 0.001, \alpha_m = 0.01, \lambda_A = 0.01, \lambda_H = 0.001,$$

$$\alpha = 0.001, a = 0.95, b = 0.97, R_1 = R_2 = 1$$

$$\mu(x) = 0.9, \nu(x) = 0.8, \beta(x) = 1,$$

and taking the inverse laplace transform of (39) the availability of the system is obtained as follows:

$$P_{up}(t) = e^{-0.0231t} + 0.00095 \left(\frac{0.707e^{-t} - 1.88e^{-0.923t}}{+0.88e^{-0.023t}} \right) + \\ 0.0097(0.1734e^{-t} - 1.462e^{-0.823t} + 1.288e^{-0.023t}) \quad (47)$$

The values of $P_{up}(t)$ for different values of t are calculated from (47) and have been shown in the Table 1.

Table 1: Variation of availability with time

Time (t)	Availability [P _{up} (t)]
0	0.999716
1	0.984218
2	0.965074
3	0.944572
4	0.923734
5	0.903014
6	0.882607
7	0.862594
8	0.843005
9	0.823848
10	0.805121

Table 2: Variation of reliability with time

Time (t)	Reliability[R(t)]
0	1
1	0.98768
2	0.975421
3	0.963227
4	0.951101
5	0.939045
6	0.927063
7	0.915156
8	0.903328
9	0.89158
10	0.879914

Table 3: Variation of profit function(non repairable system) with time

Time(t)	Profit Function[H(t)] (Non Repairable System)		
	C ₂ =10.0	C ₂ =12.5	C ₂ =15.0
1	48.363598	45.863598	43.363598
2	111.065918	106.065918	101.065918
3	173.042877	165.542877	158.042877
4	232.48999	222.48999	212.48999
5	289.529999	277.029999	264.529999
6	344.540771	329.540771	314.540771
7	397.808899	380.308899	362.808899
8	449.51181	429.51181	409.51181
9	499.756104	477.256104	454.756104
10	548.609375	523.609375	498.609375

Table 4: Variation of profit function(repairable system) with time

Time (t)	Profit Function[H(t)] (Repairable System)		
	C ₂ =10.0	C ₂ =15.0	C ₂ =20.0
1	34.621096	29.621096	24.621096
2	73.362469	63.362469	53.362469
3	111.106537	96.106537	81.106537
4	147.814384	127.814384	107.814384
5	183.482086	158.482086	133.482086
6	218.121094	188.121094	158.121094
7	251.749405	216.749405	181.749405
8	284.387589	244.387589	204.387589
9	316.057129	271.057129	226.057129
10	346.779572	296.779572	246.779572

2. Reliability analysis: Setting the values $\alpha_c = 0.001, \alpha_m = 0.01, \lambda_A = 0.01, \lambda_H = 0.001, \alpha = 0.001, a = 0.95, b = 0.97$ and taking different values of t in (43) one may obtain the reliability of the system as shown in the Table 2.

Table 5: Variation of M. T. T. F. with failure rate of B11(or B12) unit

α_m	M.T.T.F.		
	$\lambda_A=0.01$	$\lambda_A=0.02$	$\lambda_A=0.03$
0.01	65.188622	40.616913	29.257174
0.02	50.058437	34.744743	26.370426
0.03	40.232311	29.952579	23.674549
0.04	33.534729	26.195274	21.355236
0.05	28.716002	23.224821	19.394199
0.06	25.094406	20.835817	17.734848
0.07	22.277441	18.880236	16.321507
0.08	20.025625	17.253395	15.107628
0.09	18.185286	15.880587	14.05611
0.1	16.653547	14.707581	13.137749

Table 6: Variation of M. T. T. F. with human failure rate

λ_H	M.T.T.F.		
	$\alpha=0.001$	$\alpha=0.005$	$\alpha=0.010$
0.001	65.188622	52.621555	42.236332
0.002	61.545139	50.173107	40.616913
0.003	58.268799	47.933498	39.113071
0.004	55.308372	45.877781	37.71312
0.005	52.621552	43.98476	36.406895
0.006	50.173103	42.236324	35.185478
0.007	47.933491	40.616913	34.041042
0.008	45.877781	39.113071	32.966671
0.009	43.98476	37.71312	31.956251
0.01	42.236324	36.406895	31.004337

Table 7: Variation of M. T. T. F. with human failure rate

λ_H	M.T.T.F.		
	$\lambda_A=0.01$	$\lambda_A=0.02$	$\lambda_A=0.03$
0.001	65.188622	40.616913	29.257174
0.002	61.545139	39.113071	28.453701
0.003	58.268799	37.71312	27.692183
0.004	55.308372	36.406895	26.969467
0.005	52.621552	35.185478	26.282713
0.006	50.173103	34.041042	25.629339
0.007	47.933491	32.966671	25.007013
0.008	45.877781	31.956251	24.413599
0.009	43.98476	31.004337	23.847164
0.010	42.236324	30.106091	23.305927

3. Profit function

Non-repairable system: Setting the values $\alpha_c = 0.001$, $\alpha_m = 0.01, \lambda_A = 0.01$, $\lambda_H = 0.001, \alpha = 0.001$, $a = 0.95, b = 0.97$, $C_1 = 50.0, C_3 = 5.0$, using (43), taking different values of C_2 and t in (45), one may obtain the variation of profit function of the system as shown in the Table 3.

Repairable system: Using (47), taking $C_1 = 50.0, C_3 = 5.0$, different values of C_2 and t in (46), one may obtain the variation of profit function of the system as shown in the Table 4.

4. M.T.T.F analysis: Setting the values $\alpha_c = 0.001, \lambda_H = 0.001, \alpha = 0.001, a = 0.95, b = 0.97$ and taking different values of λ_A in (44), one may

obtain the variations of M.T.T.F. of the system against the failure rate of unit B11 (or B12), α_m shown in Table 5.

Setting the values $\alpha_c = 0.001, \alpha_m = 0.01, \lambda_A = 0.01, a = 0.95, b = 0.97$ and taking different values of α in (44), one may obtain the variations of M.T.T.F. of the system against the human failure rate λ_H shown in Table 6.

Setting the values $\alpha_c = 0.001, \alpha_m = 0.01, \alpha = 0.001, a = 0.95, b = 0.97$ and taking different values of λ_A in (44), one may obtain the variations of M.T.T.F. of the system against the human failure rate λ_H shown in Table 7.

RESULTS

Table 1 exhibits that the operational availability of the system decreases with increase in time period.

Table 2 shows that the reliability of the system decreases as the time period increases.

Table 3 and 4 show the variation in profit function with the increase in time period for non – repairable and repairable system respectively. The series of curve in both the figures exhibits that the profit function decreases with the increase in service cost of the system.

Table 5 depicts the variation of M.T.T.F. with the failure rate of subsystem B11 (or B12). The series of curve represents that M.T.T.F. decreases as the failure rate of sub – system A increases.

Table 6 depicts the variation of M.T.T.F. with human failure rate. The series of curve represents that M.T.T.F. decreases as the failure rate of subsystem C increases.

Table 7 depicts the variation of M.T.T.F. with the human failure rate. The series of curve represents that M.T.T.F. decreases as the failure rate of subsystem A increases.

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