American Journal of Economics and Business Administration 3 (4): 586-588, 2011
ISSN 1945-5488

Forecasting the Spanish Stock Market Returns with Fractional and Non-Fractional Models

[1]Guglielmo Maria Caporale, [2]Juncal Cunado and [2]Luis A. Gil-Alana
[1]Centre for Empirical Finance, Brunel University, London, UK
[2]Department of Economics, University of Navarra, Faculty of Economics,
Edificio Biblioteca, Entrada Este, E-31080Pamplona, Spain

Abstract: Problem statement: The content of this note was to assess the forecasting accuracy of various models of the Spanish stock market returns. **Approach:** We use daily data on the IBEX 35 for the time period January 4th, 2001-March 28th, 2006 and employ both fractional and non-fractional models. **Results:** The results on the prediction errors for the out-of-sample forecasts indicate that the fractional models outperform the non-fractional ones. **Conclusion:** Standard forecasting criteria suggest that the ARFIMA (1, d, 0) model with d = -0.017 and the AR (1) coefficient equal to 0.068 is the best specification for this series. That implies that the stock market prices display a very small degree of mean reversion behavior.

Key words: Fractional integration, stock market returns

INTRODUCTION

According to the Efficient Market Hypothesis (EMH) in its weak form, asset returns incorporate all relevant information and hence, when conditioning on historical returns, future asset returns should be unpredictable (Fama, 1970). However, if asset returns display long memory, they exhibit persistent dependence between observations far away in time, which is inconsistent with EMH since past prices can help predict future prices.

This note investigates the stochastic behavior of the Spanish stock market using fractional and nonfractional models. For the former it is assumed that degree of differencing required to get I (0) stationary returns in logs is a real value, whilst the latter are specified as stationary Auto Regressive Moving Average (ARMA) models, that is, we assume that the returns are I (0).

A lot of the literature claims that stock market prices are no stationary I (1) and, therefore, that stock market returns are stationary I (0) (Shamiri and Isa, 2009). However, as Caporale and Gil-Alana (2002) and Sowell (1992) stress, the unit root tests normally employed impose too restrictive assumptions on the behavior of the series of interest, in addition to having low power. These authors suggest instead using tests which allow for fractional alternatives. The fractionally integrated models have been already used in financial time series analysis (Cheong, 2008). Here, we follow the same approach and consider the possibility that Spanish stock returns might be fractionally integrated.

MATERIALS AND METHODS

We use the Spanish stock market IBEX 35 for the time period January 4th, 2001-28th March, 2006, leaving out the last 20 observations (March 1st, 2006-March 28th, 2006) for forecasting purposes. The IBEX 35 is a value-weighted index that includes the 35 most traded stocks on the Spanish stock market. Every six months, the effective trading volumes of all stocks are analyzed in order to adjust their weights and compute the index for the following six months. The analysis is based on daily stock market closing prices and, as standard in the literature, stock returns are calculated as $100\times(\log P_t-\log P_{t-1})$, where P_t is the stock market index at closing daily dates in period t.

RESULTS

First, we estimate the order of integration in the stock market returns, in the time and in the frequency domain, assuming that the differenced process is white noise. In the time domain, we use Sowell (Peters, 1994) procedure based on maximum likelihood estimation, while in the frequency domain we use a Whittle approximation (Dahlhaus, 1989). In both cases we obtain an estimate of 0.024. This implies that the log of IBEX is I (d) with d slightly above 1(1.024). Moreover,

Corresponding Author: Juncal Cunado, Department of Economics, University of Navarra, Faculty of Economics, Edificio Biblioteca, Entrada Este, E-31080 Pamplona, Spain Tel: +34 948 425 625 Fax: +34 948 425 626

the null hypothesis of I (0) returns is rejected at the 5% significance level, though not at the 10%.

As an alternative approach, we next assume that the log IBEX is I (1) and look at different ARMA (p,q) models for the first differences (returns). We estimate models with p and q equal to or smaller than 3 and the best specification (according to various likelihood criteria) seems to be an AR (1) process, though the AR coefficient is very close to 0(0.05105). As a third possibility, we consider the case of a fractional I (d) process with the disturbances being weakly auto correlated. In such a case, the chosen model is an ARFIMA (1, d, 0), with d equal to-0.017 and the AR coefficient again very close to 0 (0.06881). Here, according to this approach, the null hypothesis of I (0) returns cannot be rejected at conventional statistical levels. Specifically, the three selected models are:

- Model A: $(1-L)^{0.024}x_t = \varepsilon_t$
- Model B: $x_t = 0.051x_{t-1} + \varepsilon_t$
- Model C: $(1-L)^{-0.017}x_t = u_t; \quad u_t = 0.068u_{t-1} + \varepsilon_t$

in all cases with white noise ε_t.

Table 1 shows the 1-20 period-ahead prediction errors for the three specifications above. It can be seen that Model B produces the lowest prediction error only 1-period ahead. In the remaining cases lower errors are obtained with the fractional models (A and C).

Next, we compare the three models in terms of various forecasting criteria. The accuracy of different forecasting methods is a topic of continuing interest and research (Ibrahim *et al*., 2009; Harvey *et al*., 1997; Makridakis and Hibon, 2000), for a summary and review of forecasting competition). Standard measures of forecast accuracy are the following: Theil's U, the Mean Absolute Percentage Error (MAPE), the Mean-Suared Error (MSE), the Root-Mean-Squared Error (RMSE), the Root-Mean-Percentage-Squared Error (RMPSE) and Mean Absolute Deviation (MAD) (Witt and Witt, 1992). Let y_t be the actual value in period t; f_t the forecast value in period t and n the number of periods used in the calculation (in our case, 20). Then:

- Theil's U: $\dfrac{\sqrt{\sum(y_t - f_f)^2}}{\sqrt{\sum(x_t - x_{t-1})^2}}$
- Mean Absolute Percentage Error (MAPE): $\dfrac{\sum\left|(x_t - f_t)/x_t\right|}{n}$
- Mean Squared Error (MSE): $\dfrac{\sum(x_t - f_t)^2}{n}$

Table 1: Prediction errors of the selected models

Period	Model A	Model B	Model C
1	0.0135523658	0.0133796312	0.0142780738
2	-0.0080015361	-0.0078249045	-0.0076481868
3	-0.0050211195	-0.0045752666	-0.0044408152
4	-0.0000518086	0.0002891424	0.0004175665
5	-0.0051789829	-0.0049274639	-0.0048039942
6	-0.0065236748	-0.0061807701	-0.0060622157
7	0.0057446342	0.0060354770	0.0061493684
8	0.0056523844	0.0061521880	0.0062618228
9	0.0057205306	0.0062327660	0.0063384972
10	-0.0016277772	-0.0008783640	-0.0007761531
11	0.0004852574	0.0009036900	0.0010026038
12	0.0009654163	0.0014172310	0.0015131503
13	0.0004531243	0.0008763280	0.0009694912
14	0.0038031118	0.0041269410	0.0042175460
15	0.0005364111	0.0009557590	0.0010440233
16	0.0008390153	0.0015239690	0.0016101056
17	-0.0042191888	-0.0038058880	-0.0037218041
18	0.0035679138	0.0040401390	0.0041223331
19	-0.0099241473	-0.0096495660	-0.0095692249
20	-0.0058341022	-0.0055391480	-0.0054604956

Table 2: Forecasting criteria with a time horizon of 20 periods

	Model A	Model B	Model C
MAPE	0.8941656477	1.0381058509	0.9990610372
MSE	0.0000311247	0.0000318911	0.0000308051
RMSP	0.3961572101	0.4788347112	0.0000000001
RMSE	0.0055789575	0.0056472281	0.0055502341
MAD	0.0043851253	0.0045203738	0.0044657316
U Theil	0.5969771102	0.5191564002	0.5014857370

- Root-Mean-Percentage-Squared Error (RMSP): $\sqrt{\dfrac{\sum(x_t - f_t)^2/f_t}{n}}$
- Root-Mean-Squared Error (RMSE): $\sqrt{\dfrac{\sum(x_t - f_t)^2}{n}}$
- Mean Absolute Deviation (MAD): $\dfrac{\sum|x_t - f_t|}{n}$

Table 2 shows the results based on the above criteria. We note that, according to the MAPE and MAD, the pure fractional model (Model A) seems to be the best specification. However, based on the other criteria, the fractional auto regression (Model C) appears to be the most appropriate one. In any case, these statistical criteria indicate that the fractional models (with or without weak autocorrelation) outperform the non-fractional model B in all cases.

DISCUSSION

The results presented so far as based on criteria which are purely descriptive devices. Several statistical tests for comparing different forecasting models are now available. One of them, widely employed in the time series literature, is the asymptotic test for a zero expected loss differential due to (Diebold and Mariano, 1995).

Table 3: Modified DM statistic: 20-step ahead forecasts

	Model A	Model B	Model C
Model A	X	-1.040	-1.092
Model B	X	X	-1.034
Model C	X	X	X

Harvey *et al*. (1997) note that the DM test statistic could be seriously over-sized as the prediction horizon increases and therefore provide a modified Diebold-Mariano test statistic given by:

$$M - DM = DM\sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}}$$

Where:

DM = The original Diebold-Mariano statistic
h = The prediction horizon
n = The time span for the predictions

Using the M-DM test statistic based on the RMSE loss function, we further evaluate the relative forecast performance of the three models by making pair wise comparisons considering a 20-period horizon. The results are shown in Table 3. The evidence points out in favor of Model C as the best specification though none of the three statistics are statistically significant. The same happens when other loss functions are employed.

CONCLUSION

In this note we have examined the forecasting ability of various fractional and non-fractional models to describe the stochastic behaviour of the Spanish stock market returns. The results show that the fractional models outperform the non-fractional one in practically all cases. Moreover, the fact that the order of integration is found to be slightly different from zero suggests that there is some degree of forecast ability in the stock market returns, which can be seen as evidence against the Efficient Market Hypothesis (Fama, 1970) and rather in line with the Fractional Market Hypothesis of (Peters, 1994).

REFERENCES

Caporale, G.M. and L.A. Gil-Alana, 2002. Fractional integration and mean reversion in stock prices. Q. Rev. Econ. Financ., 42: 599-609. DOI: 10.1016/S1062-9769(00)00085-5

Cheong, C.W., 2008. Volatility in Malaysian stock market: An empirical study using fractionally integrated approach. Am. J. Applied Sci., 5: 683-688. DOI: 10.3844/ajassp.2008.683.688

Dahlhaus, R., 1989. Efficient parameter estimation for self-similar processes. Ann. Stat., 17: 1749-1766. DOI: 10.1214/aos/1176347393

Diebold, F.X. and R.S. Mariano, 1995. Comparing predictive accuracy. J. Bus. Econ. Stat., 13: 253-263.

Fama, E.F., 1970. Efficient capital markets. A review of theory and empirical work. J. Financ., 25: 383-417.

Harvey, D., S. Leybourne and P. Newbold, 1997. Testing the equality of prediction mean squared errors. Int. J. Forecast., 13: 281-291. DOI: 10.1016/S0169-2070(96)00719-4

Ibrahim, M.Z., R. Zailan, M. Ismail and M.S. Lola, 2009. Forecasting and time series analysis of air pollutants in several area of Malaysia. Am. J. Environ. Sci., 5: 625-632. DOI: 10.3844/ajessp.2009.625.632

Makridakis, S. and M. Hibon, 2000. The M3-competition: Results, conclusions and implications. Int. J. Forecast., 16: 451-476. DOI: 10.1016/S0169-2070(00)00057-1

Peters, E.E., 1994. Fractal Market Analysis: Applying Chaos Theory to Investment and Economics. 1st Edn., John Wiley and Sons, New York, ISBN: 0471585246, pp: 315.

Shamiri, A. and Z. Isa, 2009. Modeling and forecasting volatility of the Malaysian stock markets. J. Math. Stat., 5: 234-240. DOI: 10.3844/jmssp.2009.234.240

Sowell, F., 1992. Maximum likelihood estimation of stationary univariate fractionally integrated time series models. J. Econ., 53: 165-188. DOI: 10.1016/0304-4076(92)90084-5

Witt, S.F. and C.A. Witt, 1992. Modeling and Forecasting Demand in Tourism. 1st Edn., Academic Press, San Diego, ISBN: 0127607404, pp: 195.