# Process is Unreliable and Quantity Discounts Supply Chain Integration Inventory Model 

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#### Abstract

This paper mainly focuses on the discussion of the best economic production quantity of process unreliable and defective items reproduce of the production system. In today's production and manufacturing schedule, the quality of the supply chain often determines the efficiency of a company's operations. However, the traditional method of solving the problem of economic production quantities, mostly assumes that perfect production process does not appear defective items and backorder situation. The suppliers order to compensate for the inconvenience caused by defective products to the buyers, used quantity discounts as a way to compensate the buyers. Therefore, our proposed system based upon the production of the finished goods inventory system. In the process of reproduce of defective products, we will pick out defective products, which cannot reproduce and scrap them; the remaining defective products will be repaired and sent back to the buyer. Through this study, some numerical examples available in this study are provided and the results show that our proposed system can be applied as the usable tool.


Keywords: Quantity Discounts, The Process is Unreliable, Inventory Model

## Introduction

Inventory strategy is very important for firms, which maintain advantages in production and logistics parts. The comprehensive inventory system can achieve the best level of service and reduce manufacturing and inventory costs and maximize profits. Previous studies usually built the traditional integrated inventory model in the perfect production processes without defective products. However; in practice, the defective products are unavoidable by human errors, mechanical failures and other reasons. Therefore, this study commits to sort out what the defective rate impact of the costs for buyers and sellers and also reduce losses arising from defective products. To recover customer groups, this study envisages some remedies in the end. Suppliers commonly used quantity discount as concessions to attract buyers. However, this study considers that quantity discount is used to compensate the buyer to purchase the loss caused by defective products.

This paper presents an integrated supply chain inventory model which includes the process unreliable
and quantity discounts with the consideration of minimizing the total cost of the buyer and seller. Also, this paper assumes that the production process will produce a certain number of defective products, the buyer checks out the defective products and returned to the seller to repair and the vendor will provide a discount. To find the minimum total cost, must determine the optimal order quantity $(\mathrm{Q})$ and delivery times per production cycle ( n ), so taking first and second-order partial derivative of EK ( $\mathrm{n}, \mathrm{Q}$ ) with respect to Q and n , this paper obtains $n$ and Q's extreme value, because delivery times (n) is an integer, this study used an iterative method to calculate the optimal solution of $n$ and Q as well as the minimum total costs.

After obtaining minimum total cost, this study applies 4 parameters (screening rate (X), annual demand (D), percentage of the defective products $(\mathrm{k})$, production rate (P)) in doing sensitivity analysis with EK ( $\mathrm{n}, \mathrm{Q)}$ ) and shows the effect of 4 parameters' slightly changed. Subsequently, this paper will apply the experimental data with mathematical software for $\mathrm{Q}, \mathrm{D}$ and the EK to assess the three-dimensional map.

Starting from the previous lecture reviews, Porteus (1986) was the first researcher that incorporated the impact caused by defective products into basic EOQ model. Based on this, we know the importance of the unreliable process's impact. Lee and Rosenblatt (1987) considered the test during production runs faster than the traditional EOQ model was tested out and repair. Schwaller (1988) extended EOQ models to conform to the real-life environment of inventories by adding the assumptions that defectives of a known proportion were present in the incoming lots. Zhang and Gerchak (1990) proposed a joint lot sizing and inspection policy under an EOQ model, which shows inspection of importance in EOQ model. Then, Cheng (1991) assumed the relationship between the unit production cost and demand may be present in some cases, proposed demand and the unit cost of production and nonperfect process model EOQ. He developed this inventory decision problem as a Geometric Programming (GP) and solved to obtain a closed form of the optimal solution. Ben-Daya and Hariga (2000) considered the problem about impact of the imperfect process to make a model and assumed that the start of the production facility is in a state of perfect quality, but the facilities will deteriorate over time and at a random time change to the uncontrolled state, began to produce defective products. Salameh and Jaber (2000) assumed production and inventory situation, items, or products are not perfect quality. Defective, unwanted products can be used in other fewer restrictive procedures and acceptance control production and inventory situation with the consideration of poor-quality items at the end of the inspection process should be sold out. Goyal and Cardenas-Barron (2002) developed a model to determine the total profits per unit time and purchase products from suppliers EOQ and proposed the method of determined EPQ and defective products. Huang (2004) proposed under JIT manufacturing environment to develop a model to determine defective items held by the seller and the buyer of the best integrated inventory strategy. Huang (2004) proposed model is built on check defective products during continuous consumption of inventory and the checking out items will be reimbursed.

The discussion of this paper is to carry out the changes of inventory quantity discount model and processes unreliable situation and found out the most suitable order quantity from buyers and sellers in order to achieve the total cost of both minimized.

In today's highly competitive global markets; many marketing tactics and manufacturers are applying price discounts to attract consumers. Nason and Della Bitta (1983) proposed quantity discount which indicated that when consumers buy products with a certain number,
retailers will offer lower unit price to the buyer as the discount. Good quantity discounts can make both suppliers and buyers satisfied and might increase the orders. On the other hand, manufacturers can reduce inventory costs and set-up costs. Monahan (1984) discussed a problem from the vendor's point of view to construct an optimal quantity discount table. Lal and Staelin (1984) developed a strategy was conducive to the buyer for the best price discounts. Weng and Wong (1993) investigated some managerial insights related to using the all-unit quantity discount policies under various conditions like one buyer or multiple buyers, constant or price-elastic demand, the relationship between the supplier's production schedules or ordering policy and the buyers' ordering sizes and the supplier either purchasing or manufacturing the item. Weng (1995) proposed a viewpoint of the seller to reduce operating costs and increase buyer demand, consider the price-sensitive demand along with quantity discount policy. Chakravarty and Martin (1988) provided the vendor with the means for optimally determining both the discount price and the replenishment interval under periodic review for any desired joint savings-sharing scheme between the seller and the multiple-buyer (s). Munson and Rosenblatt (1988) proposed a third-level quantity discount with supply chain and fixed demand rate. Yang (2004) proposed a quantity discount pricing strategy was necessary to entice the buyer to accept the alliance. Wang (2005) extended traditional quantity discounts that are based solely on buyers' individual order size to discount policies that are based on both buyers' individual order size and their annual volume. They showed that discount policies are able to achieve nearly optimal system profit and, hence, provide effective coordination. Li and Liu (2006) developed a model that explains how to use quantity discount policy in order to achieve the supply chain coordination, considering only selling one product with multi-cycle and the probability of customer's demand for the buyer and seller system and suggest that the combination in a mutually acceptable quantity discount profit exceeded the sum of the profits from each other in the case of decentralized decision-making. Qin et al. (2007) established a supply chain which contains quantity discounts and price-sensitive demand. Lin and Ho (2011) let the buyers adjust their retail price based on the purchasing cost, which will influence the customer demand as a result. Yang et al. (2010) established an inventory model for a retailer in a supply chain when a supplier offers either a cash discount or a delay payment linked to ordering quantity. Lin and Lin (2014) developed a model about defective products and quantity discounts. The purpose was to find optimal
pricing and ordering strategy; the analysis is based on the buyer's order quantity. Zhang and Wu (2014) proposed Multiple Objective Decision Making (MODM) model considered the bi-fuzzy environment and quantity discount policy and quantity discount is an important factor in their study. Tominaga et al. (2008) developed the production planning model for multiple companies and examined the bullwhip effect caused by the inventory control method of safety stock. Yang et al. (2013) investigated the backorder rate inventory problem with variable lead time in which can be decomposed into several components, with a crashing cost function for the respective reduced lead time.

In the past, most studies have mostly focused on price promotions, discounts and prices strategies, because those business strategies can save money and increase profit directly. However, it has ignored the wrong quantity discount policy may have a negative impact on the profit between the buyers and the sellers. Therefore, this study determines the quantity discounts based on the defective rate, not widely discussed to reach quantity in order to get discounts in the past.

## Materials and Methods

To establish the proposed model, the following notations are used and some assumptions are made throughout this study.

## Notations

- $S_{v}$ : Set up cost for the vendor; \$/time
- $Q:$ Each time the number of transported from the buyer; pcs/times
- $\quad P$ : Production rate; pcs/year
- $\quad R$ : Recovery cost for the vendor; $\$ /$ month
- $L$ : Maintenance cost for the vendor; $\$ /$ month
- $n$ : Number of deliveries each production cycle; times
- $h v$ : Holding cost for the vendor; $\$ /$ month
- $\quad V$ : Warranty cost for the vendor; $\$ /$ month
- $Y$ : The percentage of defective products, as random variables
- $Q_{r}$ : Manufacturing cost for vendor; $\$ /$ month
- $S_{b}$ : Order cost for the buyer; $\$ /$ time
- $F$ : Transportation cost per shipment; \$/trip
- $D$ : Annual demand, $P>D$; pcs/year
- $h_{b}$ : Holding cost for buyer; $\$ /$ month
- $d$ : Screening cost for buyer; $\$ /$ month
- $X$ : Screening rate; pcs/year
- $\sigma$ : Discount rate. $; \sigma=m * Y^{*} k$, punishment multiples $(m)$ is determined by the seller themselves
- B: Purchase cost for buyer; $\$ /$ month
- $\quad T$ : Transporting each successive time intervals
- $T_{c}$ : Cycle time. $T_{c}=n^{*} T$
- $k$ : Percentage of defective products cannot be repaired percentage
- EK: Expected annual integrated total cost


## Assumptions

- This paper is based on single vendor and single buyer for single item
- The production rate is finite
- Shortages are not allowed
- Because of shortages are not allowed, non-defective product's production
- Rate must higher than buyer's demand
- Quantity discount and defective rate has direct relation
- Returned defective production will be repaired, but not fully repaired
- When buyer's inventory remaining $Q / 2$, whole products must be inspected, defective products must be picked up and sent back to the vendor
- Quantity discount have a restriction, because vendor's cost
(warranty $\operatorname{cost}(v)$, manufacturing $\operatorname{cost}(Q r)$, for vendor's discount $(\sigma B)$ ) can't more than buyer's purchase $\operatorname{cost}(B)$; otherwise, the vendor doesn't have profits:

$$
V+\frac{Q_{r}}{D}+\sigma B<B=\sigma<1-\frac{\left(\frac{Q_{r}}{D}+V\right)}{B}
$$

- The discount rate set to be $\sigma=m * Y * k$, the higher the vendor's defective product is, the higher the discount rate to the buyer next time, $m$ is magnification, decided by vendors themselves, the study assumed to be 100


## Vendor's Cost

Figure 1 shows the inventory pattern of the vendor. Vendor's cost $=$ set up cost + transportation cost + manufacturing cost + recovery cost + maintenance cost + holding cost Equation 1:

$$
\begin{align*}
& T C_{V}(\mathrm{Q}, n)= \\
& S_{V} * \frac{D}{n(1-k Y) Q}+(1+2 Y-k Y) \\
& * F * \frac{D}{(1-k Y) Q}+Q_{r} * D  \tag{1}\\
& +R^{*} k Y D+L^{*} Y^{* D} \\
& +h_{V}\left[\frac{n-1}{2}+\frac{(2-n) D}{2 p(1-k Y)}\right] Q
\end{align*}
$$

Transportation cost's derivation for vendor:

$$
\begin{aligned}
& =F * \frac{Q+Y Q+(1-k) Y Q}{Q} * \frac{D}{(1-k Y) Q} \\
& =F^{*} \frac{Q+2 Y Q+k Y Q}{Q} * \frac{D}{(1-k Y) Q} \\
& =F^{*}(1+2 Y-k Y) * \frac{D}{(1-k Y) Q}
\end{aligned}
$$

Holding cost's derivation for vendor:

$$
\begin{aligned}
& =\frac{\left\{\left[n Q\left(\frac{Q+T n p-T p}{p}\right)-\frac{n^{2} Q^{2}}{2 p}\right]-\frac{n^{2} T Q-n T Q}{2}\right\} * h_{V}}{n T} \\
& =\frac{\left(\frac{2 n Q^{2}+2 T n^{2} Q p-2 n Q T p-n^{2} Q^{2}}{2 p}-\frac{n^{2} T p Q-n T p Q}{2 p}\right) * h_{V}}{n T} \\
& =\left(\frac{2 Q^{2}-n Q^{2}}{2 p T}+\frac{2 n Q-2 Q-n Q+Q}{2}\right) * h_{V} \\
& {\left[\frac{(2-n) Q^{2}}{2 p T}+\frac{(n-1) Q}{2}\right] * h_{V}}
\end{aligned}
$$

Where Equation 2:

$$
\begin{align*}
& T=\frac{(1-k Y) Q}{\mathrm{D}} \\
& H C_{V}=h_{V}\left[\frac{n-1}{2}+\frac{(2-n) D}{2 p(1-k Y)}\right] Q \tag{2}
\end{align*}
$$

## Buyer's Cost

Figure 2 is the buyer's cost model diagram, after ordering, inventory consumption and products inspection will ongoing simultaneously until all defective products are detected and returned to the vendor for repair, repaired defective products will be returned to the buyer and continued consumption to exhaust, the buyer will continues to order the next batch of goods.

Figure 3 is the buyer's cost model diagram. It is unable to determine when the repaired defective products will be sent back to buyer. Therefore, this study sets the send back to buyer's timing in the end of inventory consumption. Based on this, this paper not only modifies the model structure, but also saves inventory holding cost more effectively.

Buyer's cost $=$ order cost + screening cost + purchase cost + warranty cost + holding cost:

$$
\begin{align*}
& T C_{B}(\mathrm{Q}, n)= \\
& S_{B} * \frac{D}{n(1-k Y) Q}+d * X+B(1-\sigma) * D+V^{*} D  \tag{3}\\
& h_{B} * \frac{1}{4}\left[\frac{\mathrm{D}}{(X+D)(1-k Y)}+\frac{2(1-k)^{2} Y^{2}-Y+1}{1-k Y}\right] Q
\end{align*}
$$

Holding cost's derivation for buyer:
$H C_{B}=$
$h_{\mathrm{B}} *\left\{\begin{array}{l}\left(\begin{array}{l}\left.\mathrm{YQ} * \frac{\mathrm{Q}}{2(\mathrm{X}+\mathrm{D})}\right) * \frac{1}{2} \\ +(1-Y) \mathrm{Q} *\left(\frac{Q}{2(X+D)}+\frac{Q}{2 D}\right) * \frac{1}{2} \\ +(1-k) Y Q * \frac{(1-k) Y Q}{D} * \frac{1}{2}\end{array}\right\} / \mathrm{T} \text {. } 1 .\end{array}\right.$
$=h_{\mathrm{B}} * \frac{1}{2} *\left[\begin{array}{c}\frac{\mathrm{Y}}{2(\mathrm{X}+\mathrm{D})} \mathrm{Q}^{2}+\frac{(1-\mathrm{Y})}{2(\mathrm{X}+\mathrm{D})} \mathrm{Q}^{2} \\ +\frac{(1-\mathrm{Y})}{2 \mathrm{D}} \mathrm{Q}^{2}+\frac{2(1-k)^{2} Y^{2}}{2 D} Q^{2}\end{array}\right] / \mathrm{T}$
$=h_{B} * \frac{1}{2}\left[\frac{Y+1-Y}{2(X+D)}+\frac{1-Y+2(1-k)^{2} Y^{2}}{2 D}\right] Q^{2} / \mathrm{T}$
$=h_{B} * \frac{1}{4}\left[\frac{1}{X+D}+\frac{2(1-k)^{2} Y^{2}-Y+1}{D}\right] Q^{2} / \mathrm{T}$
And:

$$
\begin{aligned}
& \mathrm{T}=\frac{(1-k Y) Q}{D} \\
& H C_{B}=h_{B} * \frac{1}{4}\left[\frac{\mathrm{D}}{(X+D)(1-k Y)}+\frac{2(1-k)^{2} Y^{2}-Y+1}{1-k Y}\right] Q
\end{aligned}
$$

## The Solution Procedure

From 3.1, 3.2, the total cost of both, $E K(Q, n)$ :

$$
\begin{aligned}
& E K(\mathrm{Q}, n)=T C_{V}+T C_{B} \\
& =S_{V} * \frac{D}{n(1-k Y) Q}+(1+2 Y-k Y) F \\
& * \frac{D}{(1-k Y) Q}+Q_{r} * D+R * k Y D+L^{*} Y * D \\
& +h_{V}\left[\frac{n-1}{2}+\frac{(2-n) D}{2 p(1-k Y)}\right] Q \\
& +S_{B}^{*} \frac{D}{n(1-k Y) Q}+d * X+B(1-\sigma) D+V^{*} D+h_{B} \\
& * \frac{1}{4}\left[\frac{\mathrm{D}}{(X+D)(1-k Y)}+\frac{2(1-k)^{2} Y^{2}-Y+1}{1-k Y}\right] Q
\end{aligned}
$$



Fig. 1. Inventory pattern of the vendor


Fig. 2. Inventory pattern of the buyer


Fig. 3. Inventory pattern of the buyer

By taking second-order partial derivative of $E K(n, Q)$ with respect to $Q$, which is a convex function in $Q$ for $Q>0$. Taking the derivative of Equation with respect to $Q$ :

$$
\begin{align*}
& \frac{d E K(\mathrm{Q}, n)}{d Q}= \\
& -S_{V} * \frac{D}{n(1-k Y) Q^{2}}-S_{B} * \frac{D}{n(1-k Y) Q^{2}}  \tag{5}\\
& -(1+2 Y-k Y) F^{*} \frac{D}{(1-k Y) Q^{2}}+H_{V}+H_{B}
\end{align*}
$$

Where:

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{V}}=h_{V} *\left[\frac{n-1}{2}+\frac{(2-n) D}{2 p(1-k Y)}\right] \\
& \mathrm{H}_{\mathrm{B}}=h_{B} \\
& * \frac{1}{4}\left[\frac{\mathrm{D}}{(X+D)(1-k Y)}+\frac{2(1-k)^{2} Y^{2}-Y+1}{1-k Y}\right] \\
& \text { let } \frac{d E K(\mathrm{Q}, n)}{d Q}=0
\end{aligned}
$$

Which yields:

$$
\begin{equation*}
Q^{*}=\sqrt{\frac{\left[S_{V}+S_{B}+n(1+2 Y-k Y) F\right] D}{n(1-k Y)\left(H_{V}+H_{B}\right)}} \tag{6}
\end{equation*}
$$

In order to know the effect of m for $E K(n, Q)$, so taking the derivative of $E K(n, Q)$ with respect to $Q$ :

$$
\begin{align*}
& \frac{d E K(\mathrm{Q}, n)}{d n}= \\
& -S_{V} * \frac{D}{(1-k Y) Q n^{2}}+h_{V} *\left[\frac{Q}{2}+\frac{-Q D}{2 p(1-k Y)}\right] \\
& -S_{B} * \frac{D}{(1-k Y) Q n^{2}}=0  \tag{7}\\
& n=\sqrt{\frac{\left(S_{V}+S_{B}\right) * D}{(1-k Y) Q^{*} h_{V} *\left[\frac{Q}{2}-\frac{Q D}{2 p(1-k Y)}\right]}} \\
& =\sqrt{\frac{2 p D(1-k Y)\left(S_{V}+S_{B}\right)}{h_{V} * Q^{2}(1-k Y)[p(1-k Y)-D]}}
\end{align*}
$$

Table 1. Notation for the numerical example

| $D=1000 \mathrm{pcs} /$ year | $S_{v}=2000 \$ /$ set-up | $S_{b}=300 \$ / \mathrm{cycle}$ |
| :--- | :--- | :--- |
| $X=1000 \mathrm{pcs} /$ year | $h_{v}=1 \$$ unit | $H_{b}=3 \$ / \mathrm{pcs}$ |
| $P=2000 \mathrm{pcs} /$ year | $V=2 \$ / \mathrm{pcs}$ | $d=0.5 \$ / \mathrm{pcs}$ |
| $Y=0.01$ | $Q_{r}=10 \$ / \mathrm{pcs}$ | $B=20 \$ / \mathrm{pcs}$ |
| $K=0.3$ | $F=750 \$ /$ trip |  |
| $\sigma=m * Y * k=100 * Y * k=0.3$ | $R=1 \$ / \mathrm{pcs}$ |  |

Table 2. Numerical results

| $n$ | $Q$ | $E K$ |
| :--- | :--- | :--- |
| 1 | 1496.513 | 30638.499 |
| 2 | 1087.924 | 30059.911 |
| 3 | 905.677 | 29920.564 |
| 4 | 795.665 | 29905.715 |
| 5 | 719.562 | 29941.823 |

Based on our proposed example and solution procedure, we can see the simple results which indicate that when $n=4$, the minimum of $E K, n^{*}=4, Q^{*}=795.665, E K^{*}=29905.715$ (please see Table 2)

Table 3. Sensitivity analysis of each factor

|  | k | EK | X | EK |
| :--- | :--- | :--- | :--- | :--- |
| $-25 \%$ | 0.225 | 31404.04 | 750 | 29823.20 |
| $-20 \%$ | 0.240 | 31104.38 | 800 | 29838.80 |
| $-15 \%$ | 0.255 | 30804.70 | 850 | 29854.89 |
| $-10 \%$ | 0.270 | 30505.04 | 900 | 29871.43 |
| $-5 \%$ | 0.285 | 30205.38 | 950 | 29888.37 |
| $0 \%$ | 0.300 | 2995.70 | 1000 | 29905.70 |
| $5 \%$ | 0.315 | 29606.04 | 1050 | 29923.40 |
| $10 \%$ | 0.330 | 29306.38 | 1100 | 29941.43 |
| $15 \%$ | 0.345 | 29006.71 | 1150 | 29959.77 |
| $20 \%$ | 0.360 | 28707.04 | 1200 | 29978.39 |
| $25 \%$ | 0.375 | 28407.38 | 1250 | 29997.29 |
|  | D | EK | EK |  |
| $-25 \%$ | 750 | 22994.56 | 1500 | 29769.97 |
| $-20 \%$ | 800 | 24384.40 | 1600 | 29804.43 |
| $-15 \%$ | 850 | 25770.17 | 1700 | 29831.54 |
| $-10 \%$ | 900 | 27152.16 | 1800 | 29861.08 |
| $-5 \%$ | 950 | 28530.61 | 1900 | 29884.64 |
| $0 \%$ | 1000 | 29905.70 | 2000 | 29905.70 |
| $5 \%$ | 1050 | 31277.66 | 2100 | 29924.66 |
| $10 \%$ | 1100 | 32646.62 | 2200 | 29941.79 |
| $15 \%$ | 1150 | 34012.74 | 2300 | 29957.36 |
| $20 \%$ | 1200 | 35376.14 | 2400 | 29971.57 |
| $25 \%$ | 1250 | 36736.95 | 2500 | 29984.59 |



Fig. 4. Sensitivity analysis of each factor


Fig. 5. Three-dimensional Figure ( X axis is $\mathrm{Q}, \mathrm{Y}$ axis is $\mathrm{D}, \mathrm{Z}$ axis is EK )

## Discussion

## Sensitivity Analysis

$k$ has a considerable impact on the EK from above discussion, so this paper taking sensitivity analysis for $k$ to compare with $X, D$ and $P$. Table 3 shows the sensitivity analysis of each factor.

From Fig. 4, the total cost is directly proportional to D and P , inversely proportional to k and X , where k is a certain proportion for the total cost. It is because that the discount rate is set to $\sigma=\mathrm{m}^{*} \mathrm{Y}^{*} \mathrm{k}$, which depends on how much the vendor's defective products. In addition, Fig. 5 indicates that the threedimensional image for $\mathrm{Q}, \mathrm{D}$ and EK. It makes vendor will be more strictly controlled by the production and delivery process; thereby reducing the defective rate and has vigilante's effect.

## Conclusion

In the competitive global market, both pricing strategy and the quality often determine the orientation of the consumer. It is not a good strategy for suppliers to keep continuing cut price because suppliers might decrease profit and increase negative impacts. Therefore, the well-designed quantity discount policy is very important. In this study, we incorporated quantity discount and unreliable process into integrated inventory model. In addition, this paper assumes that the buyers get the product and start to use checks simultaneously, until all defective products inspection is completed and then returned to the vendor for repair. After the repair vendor will fix
the defective products and returned to the buyer, part of defective products can't be repaired. In the solution process, k has the certain effect for the total cost from Fig. 4, so when the value $\mathrm{K}^{*} \mathrm{Y}$ is lower, will also reduce the total cost of both, represents that fewer defective products, the greater benefit for buyer and vendor. In future studies, we can consider more factors that make this model closer to real life.

## Author's Contribution

M.F. Yang: Collecting and analyzing data of this research and contributed to writing of the manuscript.
Y. Lin: Organized the study and results and contributed to writing of the manuscript.

## Ethics

This work is from Dr. M.F. Yang's original research and has not been published elsewhere.

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