

Chaos Control in Nonlinear Systems Using the Generalized Backstepping Method

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Abstract: One of the most important nonlinear systems for checking the abilities of control methods is chaos. In this study chaos in Lorenz system was used for checking abilities of new control method. This new method to control nonlinear systems was called Generalized Backstepping method because of its similarity to Backstepping but its abilities to control more systems than Backstepping. This new method was applied to Lorenz system in three ways: 1. Stabilized states of equations. 2. Track step response. 3. Track sinusoidal response. In every way, simulations proved abilities of method. Comparing this new method with Backstepping showed that in this method, states stabilize at zero in shorter time than Backstepping and input control is more limited. So new method not only is used in more systems but also has better response.

Key words: Chaos, generalized backstepping method, Lorenz system

INTRODUCTION

Before appearance of Chaos theory, scientists supposed that if the equations of a system and the initial conditions were known, then the output of the system would be obtained. If any perturbations were observed in output of the system, then the noise input of the system or incapability recognized as the cause.

With the appearance of the new generation computers and increasing calculation power, the problem became more and more serious; since powerful computers solve nonlinear equations and draws the answer of the equations for a long time, it would leave no question for justification of the problem no longer. Therefore, many works have been done in this field in recent years^[1-9].

In this study, the Generalized Backstepping Method is used as an approach to control Chaos in Lorenz system and eventually the results of this method would be compared with the Chaos control results of Backstepping method^[10] which is based on a recursive application of Lyapunov theory.

Lorenz equations: The Chaos theory was discovered for the first time in meteorological activities by the mathematician and meteorologist of MIT called Edward Lorenz^[11]. Although the scientists were very interested in solving the problems in connection with nonlinear systems, but none of them accomplished this problem, seriously. Lorenz declared that the intensive sensitivity to initial conditions would cause in predictability of these equations in the next years.

He obtained a series of equations by simplifying the available systems which were then called Lorenz equations terminologically. His work was based on modeling heat transfer process that resulted finally in achieving Eq. 1^[4].

$$\begin{cases} \dot{x} = -10x + 10y \\ \dot{y} = -xz - y \\ \dot{z} = xy - z - R \end{cases} \quad (1)$$

In which $R = R_0 + u$ was Raleigh Number and R_0 was performance index and u was input control Signal.

If the value of u and R_0 are selected as $u = 0$ and $R_0 = 28$ then the Eq. 1 is a chaotic System and has three unstable states which are shown in the Fig. 1 and 2.

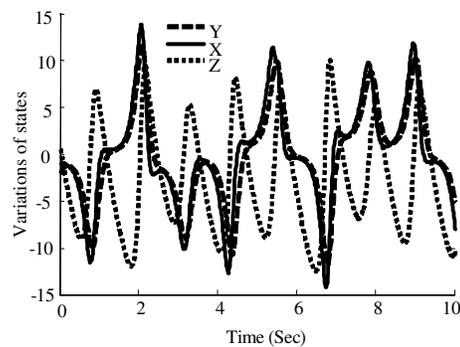


Fig. 1: Variations of states before stabilization.

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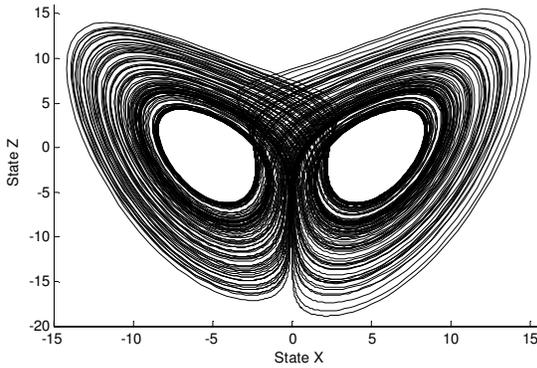


Fig. 2: X-Z Phase portrait before stabilizing

MATERIALS AND METHODS

Generalized Backstepping method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow.

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X, \eta) + g_0(X, \eta)u \end{cases} \quad (2)$$

In which $\eta \in \mathfrak{R}$ and $X = [x_1, x_2, \dots, x_n] \in \mathfrak{R}^n$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

Theorem: Suppose Eq. 2 is available, then suppose the scalar function $\Phi_i(x)$ for the i th state could be determined in a manner which by inserting the i th term for η , the function $V(X)$ would be a positive definite Eq. 3 with negative definite derivative.

$$V(X) = \frac{1}{2} \sum_{i=1}^n x_i^2 \quad (3)$$

Therefore, the control signal and also the general control Lyapanov function of this system can be obtained by Eq. 4, 5

$$u = \frac{1}{g_0(X, \eta)} \left\{ \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \Phi_i}{\partial x_j} [f_j(X) + g_j(X)\eta] - \sum_{i=1}^n x_i g_i(X) - \sum_{i=1}^n k_i [\eta - \Phi_i(X)] - f_0(X, \eta) \right\} \quad (4)$$

$$\sum_{i=1}^n x_i g_i(X) - \sum_{i=1}^n k_i [\eta - \Phi_i(X)] - f_0(X, \eta) > 0; i=1, 2, \dots, n$$

$$V_t(X, \eta) = \frac{1}{2} \sum_{i=1}^n x_i^2 + \frac{1}{2} \sum_{i=1}^n [\eta - \Phi_i(X)]^2 \quad (5)$$

Proof: The Eq. 2 can be represented as the extended form of Eq. 6.

$$\begin{cases} \dot{x}_i = f_i(X) + g_i(X)\eta ; i=1, 2, \dots, n \\ \dot{\eta} = f_0(X, \eta) + g_0(X, \eta)u \end{cases} \quad (6)$$

$V(X)$ is always positive definite and therefore the negative definite of its derivative should be examined; it means $W(X)$ in Eq. 7 should always be positive definite, so that $\dot{V}(X)$ would be negative definite.

$$\dot{V}(X) = \sum_{i=1}^n x_i \dot{x}_i = \quad (7)$$

$$\sum_{i=1}^n x_i [f_i(X) + g_i(X)\Phi_i(X)] \leq -W(X)$$

By $u_0 = f_0(X, \eta) + g_0(X, \eta)u$ and adding and subtracting $g_i(X)\Phi_i(X)$ to the i th term of Eq. 6 and 8 be obtained.

$$\begin{cases} \dot{x}_i = [f_i(X) + g_i(X)\Phi_i(X)] + g_i(X)[\eta - \Phi_i(X)] \\ \dot{\eta} = u_0 \end{cases} \quad i=1, 2, \dots, n \quad (8)$$

Now we use the following change of variable.

$$z_i = \eta - \Phi_i(X) \Rightarrow \dot{z}_i = u_0 - \dot{\Phi}_i(X) \quad (9)$$

$$\dot{\Phi}_i(X) = \sum_{j=1}^n \frac{\partial \Phi_i}{\partial x_j} [f_j(X) + g_j(X)\eta] \quad (10)$$

Therefore, the Eq. 8 would be obtained as follows:

$$\begin{cases} \dot{x}_i = [f_i(X) + g_i(X)\Phi_i(X)] + g_i(X)[\eta - \Phi_i(X)] \\ \dot{z}_i = u_0 - \dot{\Phi}_i ; i=1, 2, \dots, n \end{cases} \quad (11)$$

Regarding that z_i has n states, the u_0 can be considered with n terms, provided that the Eq. 12 would be established as follows.

$$u_0 = \sum_{i=1}^n u_i \quad (12)$$

Therefore, the last term of Eq. 11 would be converted to Eq. 13

$$\dot{z}_i = u_i - \dot{\Phi}_i(X) = \lambda_i \quad (13)$$

At this Stage, the control Lyapanov function would be considered as Eq. 14

$$V_t(X, \eta) = \frac{1}{2} \sum_{i=1}^n x_i^2 + \frac{1}{2} \sum_{i=1}^n z_i^2 \tag{14}$$

Which is a positive definite function. Now it is sufficient to examine negative definitively of its derivative.

$$\dot{V}_t(X, \eta) = \sum_{i=1}^n \frac{\partial V(X)}{\partial x_i} [f_i(X) + g_i(X)\Phi_i(X)] + \sum_{i=1}^n \frac{\partial V(X)}{\partial x_i} g_i(X) + \sum_{i=1}^n z_i \lambda_i \tag{15}$$

In order that the function $\dot{V}_t(X, \eta)$ would be negative definite, it is sufficient that the value of λ_i would be selected as the Eq. 16

$$\lambda_i = - \frac{\partial V(X)}{\partial x_i} g_i(X) - k_i z_i \quad ; \quad k_i > 0 \tag{16}$$

Therefore, the value of would be obtained from following equation.

$$\dot{V}_t(X, \eta) = \sum_{i=1}^n x_i [f_i(X) + g_i(X)\Phi_i(X)] - \sum_{i=1}^n k_i z_i^2 \leq -W(X) - \sum_{i=1}^n k_i z_i^2 \tag{17}$$

Which indicates that the negative definitively status of the function $\dot{V}_t(X, \eta)$.

Consequently, the control signal function, using the Eq. 8, 10 and 11 would be converted to 18

$$u_0 = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \Phi_i}{\partial x_j} [f_j(X) + g_j(X)\eta] - \sum_{i=1}^n x_i g_i(X) - \sum_{i=1}^n k_i [\eta - \Phi_i(X)] \tag{18}$$

Therefore, using the variations of the variables which we carried out, the Eq. 4, 5 can be obtained. Now, considering the unlimited region of positive definitively of $V_t(X, \eta)$ and negative definitively of $\dot{V}_t(X, \eta)$ and the radially unbounded space of its states, global stability gives the proof.

Simulation of Lorenz system: In order to convert Lorenz equations to the general state of Eq. 2, the change variable $r = y-x$ should be carried out.

Therefore, the Eq. 1 would be converted to Eq. 19, as follows.

$$\begin{cases} \dot{r} = -10r - y + z(r - y) \\ \dot{y} = -y + z(r - y) \\ \dot{z} = -y(r - y) - z - 28 - u \end{cases} \tag{19}$$

Stabilization of the states: In order to use the theorem, it is sufficient to establish Eq. 20,21

$$\Phi_1(r, y) = \frac{y}{r - y} \tag{20}$$

$$\Phi_2(r, y) = 0 \tag{21}$$

According to the theorem, the control signal will be obtained from the Eq. 22

$$u = - \frac{y}{(r - y)^2} \dot{r} + \frac{r}{(r - y)^2} \dot{y} + r(r - y) + k_1(z - \frac{y}{r - y}) + (k_2 - 1)z - 28 \tag{22}$$

Step response tracking: Suppose, the y state would be output of the system and it would track the input response. In this case by using the change of variable $w = 1-y$ Eq. 19 would be converted to the Eq. 23

$$\begin{cases} \dot{r} = -10r + w - 1 + z(r + w - 1) \\ \dot{w} = 1 - w + z(r + w - 1) \\ \dot{z} = (w - 1)(r + w - 1) - z - 28 - u \end{cases} \tag{23}$$

By choosing Eq. 24,25 according theorem, input control signal would become as Eq. 26

$$\Phi_1(r, w) = \frac{1 - w}{r + w - 1} \tag{24}$$

$$\Phi_2(r, w) = \frac{-1}{r + w - 1} \tag{25}$$

$$u = \frac{r-1}{(r+w-1)^2} \dot{w} - \frac{w}{(r+w-1)^2} \dot{r} + z + 28 + (r+2w-1)(r+w-1) + k_1(z + \frac{w-1}{r+w-1}) + k_2(z + \frac{1}{r+w-1}) ; k_1, k_2 > 0 \tag{26}$$

Sinusoidal response tracking: Suppose the state x would track the sinusoidal response. In this case by using the change of variables like $x_1 = b \sin \omega t - r$, $x_2 = y$ and $a = z$, the Eq. 19 will be converted to Eq. 27

$$\begin{cases} \dot{x}_1 = -10x_1 - x_2 + 10\alpha\sin\omega t + \alpha\omega\cos\omega t + \eta(x_1 - x_2 - \alpha\sin\omega t) \\ \dot{x}_2 = -x_2 + \eta(x_1 - x_2 - \alpha\sin\omega t) \\ \dot{\eta} = -x_2(x_1 - x_2) + x_2\sin\omega t - \eta - 28 - u \end{cases} \quad (27)$$

In order to use theorem, the Eq. 28,29 should be established.

$$\Phi_1(x_1, x_2) = \frac{x_2 - 10\alpha\sin\omega t - \alpha\omega\cos\omega t}{x_1 - x_2 - \alpha\sin\omega t} \quad (28)$$

$$\Phi_2(x_1, x_2) = 0 \quad (29)$$

Using the theorem, the control signal will be written as the Eq. 30

$$\begin{aligned} u = & \frac{x_2 - 10\alpha\sin\omega t - \alpha\omega\cos\omega t}{(x_1 - x_2 - \alpha\sin\omega t)^2} \dot{x}_1 - x_2(x_1 - x_2) \\ & - \frac{x_1 - 11\alpha\sin\omega t - \alpha\omega\cos\omega t}{(x_1 - x_2 - \alpha\sin\omega t)^2} \dot{x}_2 + \\ & (x_1 + x_2)(x_1 - x_2 - \alpha\sin\omega t) \\ & + k_2\eta - \eta - 28 + \alpha x_2\sin\omega t + k_1 \\ & \left(\eta - \frac{x_2 - 10\alpha\sin\omega t - \alpha\omega\cos\omega t}{x_1 - x_2 - \alpha\sin\omega t} \right) \quad k_1, k_2 > 0 \end{aligned} \quad (30)$$

RESULTS

By applying control function to the Eq. 19, the Fig. 3 and 4 will represent the system states and control signal, respectively. The stabilized system phase portrait diagram is shown in Fig. 5.

Figure 6 shows that system tracks step response very well and Fig. 7 shows the input control for tracking

The simulation results are shown in the Fig. 8 and 9.

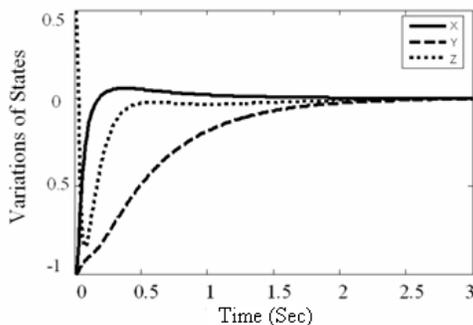


Fig. 3: x, y and z variations after stabilization

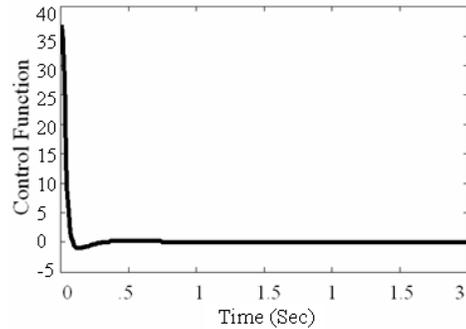


Fig. 4: input control signal

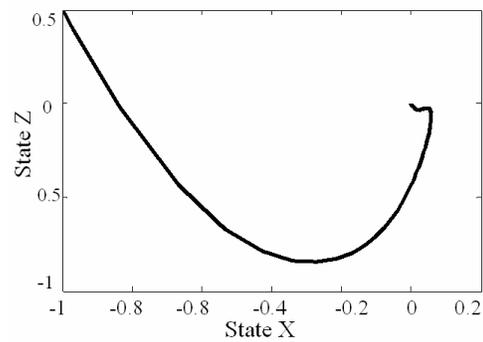


Fig. 5: X-Z Phase portrait after stabilizing

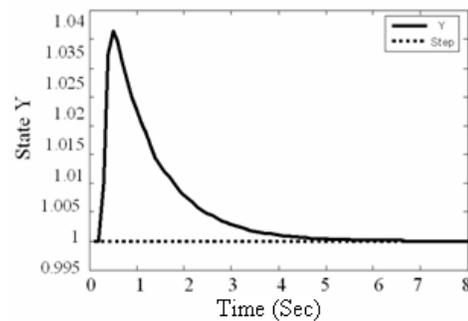


Fig. 6: Step input tracking by Lorenz system

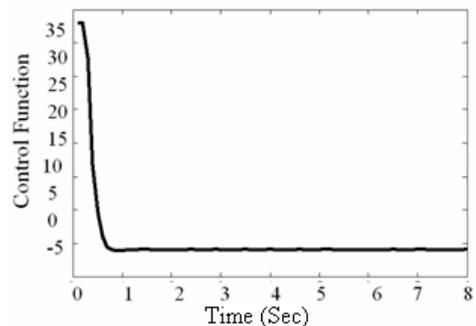


Fig. 7: Input control signal for tracking

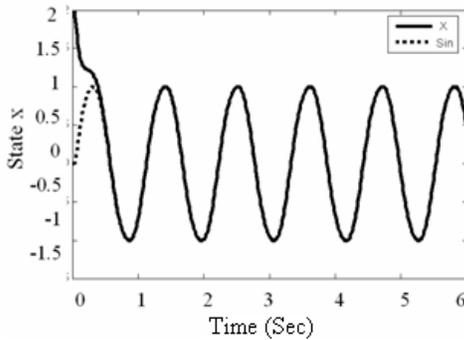


Fig. 8: Sinusoidal input tracking by Lorenz equations

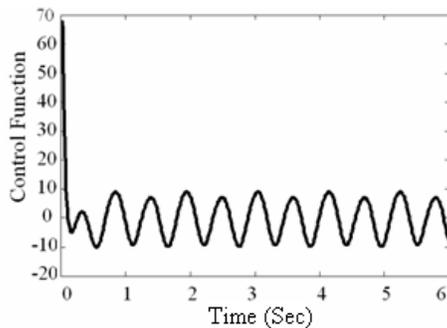


Fig. 9: Input control signal for tracking

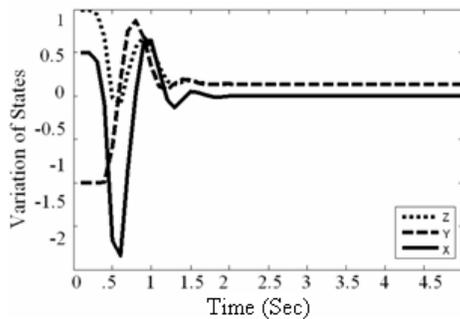


Fig. 10: Variations of states after stabilization in backstepping method

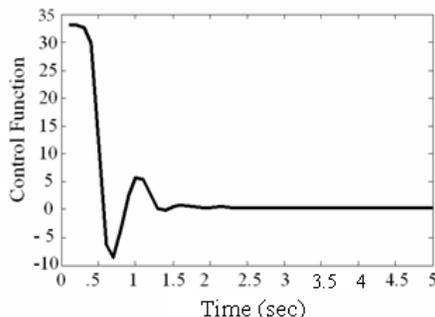


Fig. 11: Input control signal in Backstepping Method

DISCUSSION

In ref.^[10] the Backstepping method was used to control the Lorenz equations that Fig. 10 and Fig. 11 represent the simulation results, But in this study, we use Generalized Backstepping method. Now we would compare the results of the proposed method and the results in ref.^[10].

By comparing the Figs., the following results can be obtained.

- In the Backstepping Method, all dynamics would have the final value unequal to zero; while in the Generalized Backstepping Method, all dynamics would have tendency towards Zero.
- In the Generalized Backstepping Method in relation to the Backstepping Method, the system states are stabilized by a more limited control signal. Consequently, it is less possible that the control signal to be saturated
- In the Generalized Backstepping Method in relation to the Backstepping Method^[10], tracking will be accomplished in a much shorter time

Considering the results obtained from simulations, the much more efficiency of Generalized Backstepping Method in relation to the Backstepping Method will be demonstrated.

CONCLUSIONS

In this study, a new method to control nonlinear systems is presented. The method proposed which is called the Generalized Backstepping, by feed backing the dynamics of system and without eliminating the nonlinear dynamics, a controller is designed. A theorem is expressed for this method and the proof is given. Consequently, using this method, a controller is designed for the Lorenz chaotic system which is compared with the results obtained from the controller using the Backstepping Method.

The efficiency of this method is demonstrated by the accomplished simulations and comparing them with the former obtained results. Therefore, the recommended Generalized Backstepping Method in comparison with the Backstepping method not only has higher application in the real system but also has better response.

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