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Tuning of a Proportional-Integral-Derivative Controller using Multi-Objective Non Dominated Sorting Particle Swarm Optimization Applied to pH Control in Continuous Stirred Tank Reactor

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Abstract: Problem statement: Most of the control engineering problems are characterized by several, contradicting, conflicting objectives, which have to be satisfied simultaneously. Two widely used methods for finding the optimal solution to such problems are aggregating to a single criterion and using Pareto-optimal solutions. **Approach:** Non-Dominated Sorting Particle Swarm Optimization algorithm (NSPSO) based approach is used in the design of multiobjective PID controller to find the constant proportional-integral-derivative gains for a chemical neutralization plant. The plant considered in this study is highly non-linear and with varying time delay, provides a challenging test bed for nonlinear control problems. **Results:** Experimental results confirm that a multi-objective, Paretobased GA search gives a better performance than a single objective GA. **Conclusion:** Finally, the results for single objective and multiobjective optimization using NSPSO for the neutralization plant are compared. Gain scheduled PID controllers are designed from Pareto front obtained with NSPSO which exhibit good disturbance rejection capability.

Key words: Multi-objective PID controller, NSPSO, pareto optimal front, disturbance rejection, particle swarm, non dominated sorting, vector evaluated, sodium hydroxide

INTRODUCTION

Significant Research interests exists in the pH control problem because of the fact that pH processes are very difficult to control. The task of regulating the pH value in an acid-base titration process is a challenging problem. Variations in the titration curve with changes in feed conditions further complicate the dynamics of the control problem. Due to these reasons, pH control is viewed as an important benchmark for control of highly non linear processes. At an earlier stage, pH Control was done using linear conventional controllers (Nagammai *et al*., 2006) by employing cascade and feed forward controllers as proposed by Mcmillan. They suffer from the problems of robustness and load disturbances. To overcome the shortcomings, nonlinear adaptive controllers were applied to pH control problems .Later in 1990s, fuzzy logic and neural network (Loh *et al*., 1995) based modeling and control techniques were developed for the acid- base titration process(Bharathi *et al*., 2006). A fuzzy self-tuning PI controller (Jain *et al*., 2011) for a non linear process is used to tune the controller PI gains on-line by means of a parameter that results from a fuzzy inference mechanism.Combination of proportional plus integral controller (PI) controller and Fuzzy Logic Controller (FLC) is used for nonlinear control of pH neutralization process.Velocity based linearized models are proposed to modify the Internal Model Control and are applied to strongly nonlinear pH neutralization process to eliminate the steady state offsets.Genetic Algorithms (GA) are applied to pH process which searches for high performances membership and on the other hand, Fuzzy Logic Controller (FLC) manipulates the pH system.

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Some basic difficulties in the gradient methods such as their strong dependence on the initial guess can cause them to find a local optimum rather than a global one. This has led to other heuristic optimization methods, particularly Genetic Algorithms (Mwembeshi *et al*., 2004) being used extensively during the last decade. Such nature-inspired evolutionary algorithms proposed by (Deb, 2001) differ from other traditional calculus based techniques. The main difference is that GA's work with a population of candidate solutions, not a single point in search space. This helps significantly to

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avoid being trapped in local optima as long as the diversity of the population is well preserved.

In multi-objective optimization problems, (Andrey *et al*., 2005; Kumar and Nair, 2010a; 2010b) there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that as one objective function improves, another deteriorates. Therefore, here is no single optimal solution that is best with respect to all the objective functions. Instead, there is a set of optimal solutions, well known as Pareto optimal solutions which distinguishes significantly the inherent natures between single-objective and multi-objective optimization problems. V. Pareto was the French-Italian economist who first developed the concept of multi-objective optimization in economics. The concept of a Pareto front in the space of objective functions in Multi-Objective Optimization Problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space.

The early use of evolutionary search is first reported in 1960's by Rosenberg. Since then, there has been a growing interest in devising different evolutionary algorithms for MOPs. Basically, most of them are Pareto-based approaches and use the well-known non-dominated sorting procedure. In such Pareto-based approaches, the values of objective functions are used to distinguish the non-dominated solutions in the current population. Among these methods, the Vector Evaluated Genetic Algorithm (VEGA) proposed by Schaffer, Non-dominated Sorting Genetic Algorithm (NSGA) by Srinivas and Deb (1994) and Strength Pareto Evolutionary Algorithm (SPEA) by Zitzler and Thiele and the Pareto Archived Evolution Strategy (PAES) by Knowles and Corne are the most important ones. Basically, both NSGA and MOGA as Pareto-based approaches use the revolutionary non-dominated sorting procedure originally proposed by Goldberg.Brief description of Elitist Non Dominated Sorting Particle Swarm Algorithm is proposed.

This study aims at using an acid-base neutralization process of a Continuously Stirred Tank Reactor (CSTR) as a challenging test bed for examining the feasibility of solutions obtained with NSPSO. First an brief overview of pH neutralization process is presented Table 1. Aggregation based single objective optimization is then performed. Multiobjective Optimization based on NSPSO and algorithm for NSPSO is presented. Finally Gain Scheduled PID control and Concluding Remarks are presented.

MATERIALS AND METHODS

pH Neutralization process: In Industries, the process of mixing acid with base usually takes place in a large tank. The capacity of mixing vessel serves to dampen the influence of disturbances, in the form of variations in concentration and flow rate,on the pH level.Titration between acetic acid and sodium hydroxide takes place in a Continuous Stirred Tank Reactor (CSTR).

Figure 1 shows the CSTR with two input streams and an output stream.

Assumptions made:

- Volume of the Process Tank is constant
- Solution is perfectly mixed
- Chemical reactions attain chemical equilibrium instantaneously

Dynamics of the mixing process can be described by the following set of bilinear equations given by

$$v\frac{dx_a}{dt} = F_a C_a - \left(F_a + F_b\right) x_a \quad (1)$$

$$v\frac{dx_b}{dt} = F_b C_b - \left(F_a + F_b\right) x_b \quad (2)$$

Where:

Fa = the process stream (acid) flow rate in mL sec^{-1}
Ca = The concentration of the acetic acid in gmol L^{-1}
Fb = The titrating stream (base) flow rate in mL sec^{-1}
Cb = The concentration of the sodium hydroxide in gmol L^{-1}
V = The volume mixture in litres

Table 1: Process parameters

Process stream Inflow, Fa	26.6 mL sec^{-1}
Average titrating stream Inflow, Fb	3.3 mL sec^{-1}
Volume of Process tank (V)	2L
Stream concentration, Ca	0.025 g mol L^{-1}
Titrating Stream concentration, Cb	0.2 g mol L^{-1}

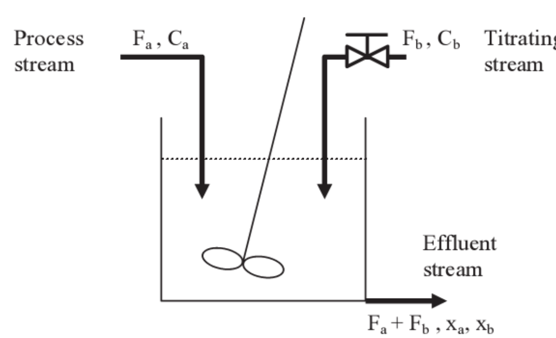


Fig. 1: pH Control using CSTR

The pH process considered is the titration reaction between a weak acid and a strong base. Acetic acid CH_3COOH) and Sodium hydroxide (NaOH) were employed as the process and titrating streams respectively.The chemical reaction that occurs when acetic acid is mixed with sodium hydroxide is governed by the following equation:

$$NaOH\ CH_3\ COOH= NaCH_3\ COO +H_2O \quad (3)$$

Acetic acid will decompose into hydrogen ion and actate ion, according to ionic dissociation theory:

$$CH_3\ COOH \rightarrow CH_3\ COO^- \quad (4)$$

Sodium hydroxide will decompose into sodium ion and hydroxyl ion:

$$NaOH = Na^+ + OH^- \quad (5)$$

The titration equation is a static relationship that maps the unit of reagents added per unit of influent to the output pH value. It is derived from the electro neutrality condition, which states that the ionic equation must be balanced in terms of sum of the charges, equating the positively charged ions with negatively charged ions.

Ionic equation for reaction between acetic acid and sodium hydroxide is obtained as:

$$[Na^+] + [H^-] = [CH_3 + COO^-] + [OH^-] \quad (6)$$

Defining $x_a = [CH_3 + COO -] + [CH_3 + COOH]$ as weak acid ionic concentration and $x_b = [Na^+]$ b base ionic concentration, the neutralization equation for the titration process is:

$$[H^+]^3 +[H^+]^2 (K_a+K_b) +[H^+]\{ K_a(x_b-x_a)- K_w\} -K_w Ka = pH=- \log 10[H^+] \quad (7)$$

$$K_a = \frac{\left[CH_3COO^-\right]\left[H^+\right]}{\left[CH_3COOH\right]} = 10^{-4.75}$$

Where:

K_a = Acid dissociation constant for acetic acid at 25°C

$K_w = \frac{\left[OH^-\right]\left[H^+\right]}{\left[H_2O\right]} = 10^{-14}$ = Ionic product of water at 25°C

The output of weak-acid strong base titration process is be derived by solving Eq. 6 and based on the solutions and Eq. 5, pH value is calculated.

Single objective optimization: The Objective functions employed are:

$$J_1 = \int_0^t t\left|e(t)\right|dt \quad (8)$$

$$J_2 = \int_0^t \left|u(t)\right|dt \quad (9)$$

Two design objectives are considered (1) minimal error (e) between reference signal and the output and (2) minimal control effort (u). Minimization of first objective provides good reference tracking and better disturbance rejection whereas minimization of second objective reduces the quantity of acid and base and thus the cost of control. J_1 is integral of time multiplied absolute error, so errors that exists for larger times are heavily penalized. J_2 is a good measure of total acid and base quantities required for control.

Objective Aggregation method is used to combine the two objectives into a single one:

$$J = J_1 + WJ_2 \quad (10)$$

Sampling time Ts is chosen to be 0.5 sec. K_p, K_i and K_d are the design parameters of PID controller.

Transfer function of PID controller is given by:

$$K(Z)K_p + \frac{K_I T_S}{z-1_S} + 2K_D \frac{z-1}{\left(T_S - 2T\right)z_c + T_S - 2T_c}$$

where, Tc = 0.5 ms is the time constant for the derivative.

Raw values of the objective functions and their importance could be considered while choosing the appropriate weight. When such information is not available, different values of W are used.

The parameters used in GA for single objective optimization are:

Population size	: 20
No. of generations	: 200
Selection	: Roulette wheel selection
Mutation Probability	: 0.05
Crossover Probability	: 0.87

Simulation results have shown that better convergence is achieved within 200 generations. 52 weights are selected for simulation and different PID values are obtained. J_1 and J_2 are computed and Fig. 2 shows the implementation results.The main reason for the gap existing in Fig. 2 is the high sensitivity of aggregated objective functions at some regions of the Pareto front.

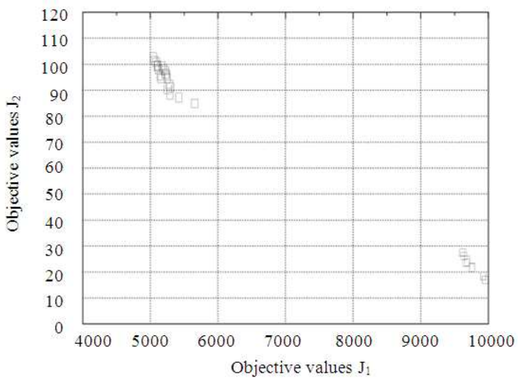


Fig. 2: Objective aggregation based optimization

Multi-objective non dominated soting particle swarm optimization: The goal of our multi-objective hybrid algorithm is to combine single-objective PSO with NSGA-II operations without loosing performance on establishing the Paretofront. The NSPSO combines the strengths of the these advanced operations (A fast non-dominated sorting approach, crowding distance ranking, elitist strategy, mutation and selection operations) with single-objective PSO search (Fig. 4). The hybrid algorithm is presented below:

Step 1: Generate an initial population P (Population size = N) and velocity for each individual (agent or particle) in a feasible space; Set the maximum speed vi max (vi max = its upper bound minus lower bound) for a variable.
Step 2: Sort the population based on the non-domination and crowding distance ranking.
Step 3: Do rank-based selection operator (Carlos and Peter, Fleming, 1993).
Step 4: Assign each individual a fitness (or rank) equal to its non-domination level (minimization of fitness is assumed).
Step 5: Randomly choose one individual as gbest for N times from the nondominated solutions and modify each searching point using previous PSO formula and the gbest:

$$v_i\,(k+1)=k\,[v_i^k+c_i \text{ x rand ( ) x } (pbest_i-s_i^k) +c_2 \text{ x rand ( ) x } (g)]$$

$$K=\frac{2}{\left|2-\varphi-\sqrt{\varphi^2-4\varphi}\right|} \quad \text{where}\varphi/=c_1+c_2,\varphi>4 \tag{11}$$

$$Si^{-k+1}=s_i^k+v_i^{-k+1} \tag{12}$$

where, rand ( ) is a random number between (0, 1). The constriction factor approach can generate higher quality solutions than the conventional PSO approach.

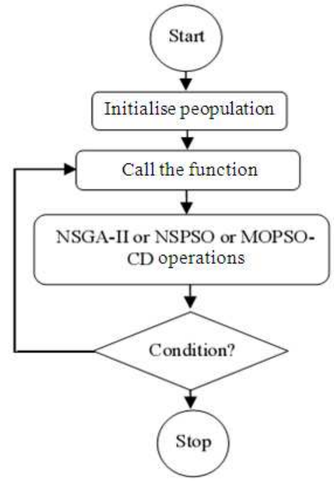


Fig. 3: Outline of optimization process

If current position outside the boundaries, then it takes the upper bound or lower bound and its velocity is generated randomly ($0 \le v_i^{k+1} \le {}_{vi}^{max}$) and multiplied by -1 so that it searches in the opposite direction.

Step 6: Do mutation operator (David, 1985).
Step 7: Combine the offspring and parent population to form extended population of size 2N.
Step 8: Sort the extended population based on nondomination and fill the new population of size N with individuals from the sorting fronts starting to the best.
Step 9: Modify the pbesti of each searching point: If current rank of the new individual (offspring) P_i^{K+1} is smaller than or equal to the previous one (parent) in R, replace the $pbest_i$ with current individual; otherwise keep the previous $pbest_i$.
Step 10: Perform steps (2-9) until the stopping criterion is met.

RESULTS AND DISCUSSION

Conventional methods of tuning of controllers are proposed by Zeigler and Nichol's.This method can be employed only for lower order linear systems. Only a single value of K_p, K_i and K_d can be obtained. In case of Multiobjective Optimization, from the Pareto front, different values of K_p, K_i and K_d can be obtained for a particular process based on different objectives.

The parameters used in NSPSO for multiobjective optimization are:

Maximum No. of Generations	: 200
Population size	: 50
Cross Over probability	: 0.85
Mutation probability	: 0.05
Crossover	: Simulated Binary

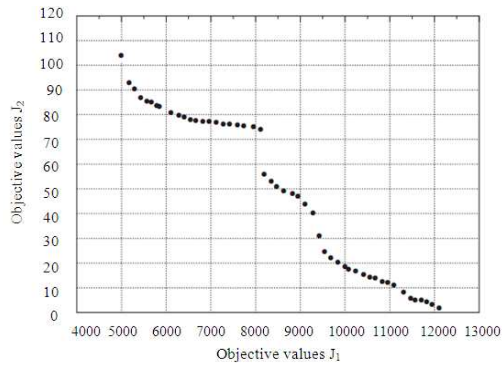


Fig. 4: Optimization results obtained with NSPSO

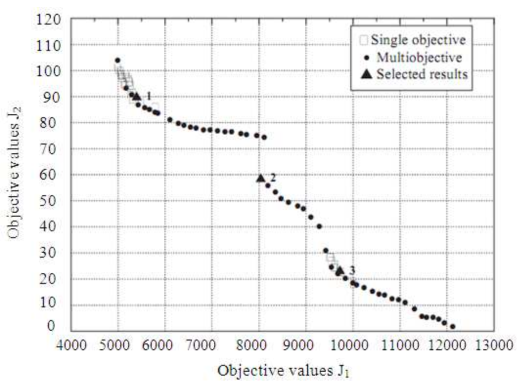
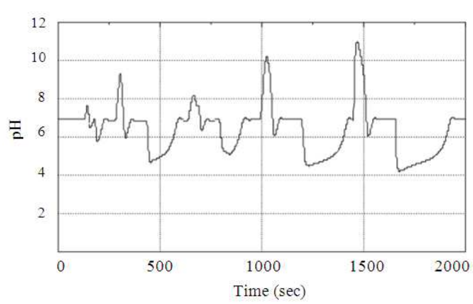
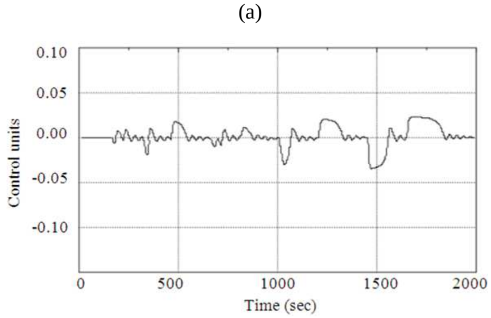


Fig. 5: Optimization results obtained with aggregation method and NSPSO and Selected points



(a)



(b)

Fig. 6: Simulation Results for selected point (Output pH for point 1)

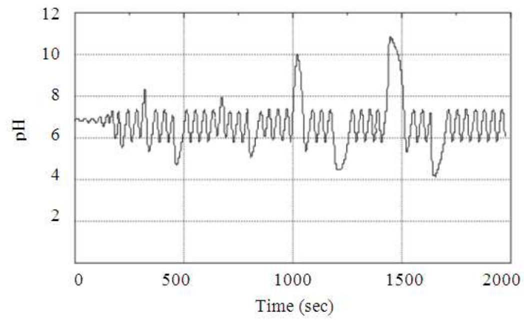
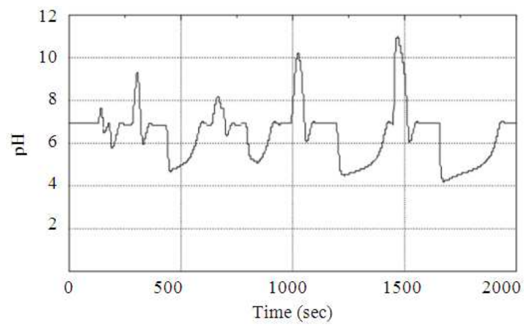
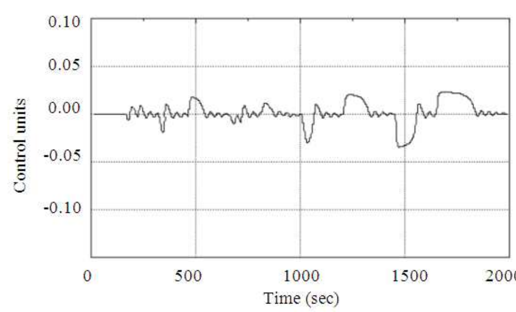


Fig. 7: Simulation results for selected point 2



(a)



(b)

Fig. 8: Optimization results obtained with aggregation method and NSPSO and Selected points

From the Pareto front shown in Fig. 5, Points 1, 2-3 shows the region where single objective and multiobjective optimization results match.

Figure 6 shows the output pH for the point 1. The following are the values obtained for point 1: $K_p = 2.77\times10^{-2}$, $K_i = 0$ and $K_d = 2.4\times10^{-3}$.

Figure 7 shows the simulation Results for selected point 2.Values obtained are $K_p = 1.57\times10^{-2}$, $K_i = 9.97\times10^{-6}$ and $K_d = 0$

Figure 8 shows the simulation Results for selected point 3.Values obtained are $K_p= 5.77\times10^{-2}$, $K_i = 0$, $K_d = 0$.

From the Pareto front shown in Fig. 3, corresponding K_p, K_i and K_d values are obtained as per the requirements of the user.

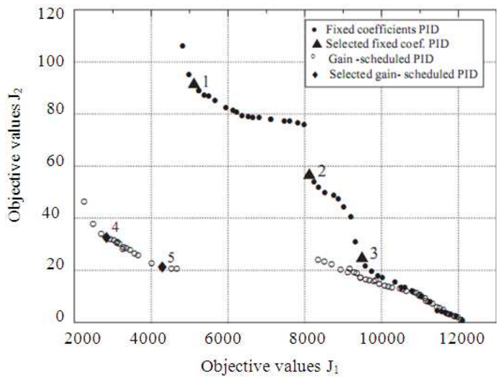


Fig. 9: Comparison of fixed gain and gain scheduled controllers

Figure 9 to obtain 50 points with aggregation methods, a total number of 50×200×20 = 200,000 closed loop simulations had to be performed. However for the same 50 points on multiobjective method only 50×200 = 10,000 simulations were needed.

Therefore NSPSO is 20 times faster than aggregation technique. It is inferred that all points in the pareto optimal set cannot be obtained with aggregation based approach.

From the results, it is obvious that gain scheduled PID controller is employed for faster disturbance rejection than fixed gain PID controller. The better performance of gain scheduled controller in rejecting acid disturbances is caused by the larger control signal (Large amplitude for a short period of time). From the simulation results, it is inferred that NSPSO outperforms other algorithms in quick disturbance rejection capability of Gain scheduled PID Controllers.

CONCLUSION

In this study designing of PID parameters with multiobjective Non Dominated Sorting Particle Swarm Optimization (NSPSO) for control of pH in a CSTR has been proposed. The main objective functions to be minimized are integral of time multiplied absolute error and control effort. Aggregation based approach and NSPSO have been used to design a fixed gain PID controller for non linear chemical pH process. The optimization solution results are a set of near optimal trade-off values which are called the Pareto front or optimality surfaces. Pareto front enables the operator to choose the best compromise or near optimal solution that reflects a trade-off between key objectives. In this study more values of PID controller parameters (K_p, K_i and K_d) can be obtained from a single Pareto Optimal front, so the designer has the flexibility to select a single solution based on the two objectives. The simulation results show that NSPSO is capable of regulating pH level over a wide range with minimal overshoot. Gain scheduled PID controller design offers faster disturbance rejection capability.

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