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Numerical Investigation of Incompressible Fluid Flow through Porous Media in a Lid-Driven Square Cavity

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Abstract: Problem statement: Investigation of fluid flow behavior through porous media in a lid-driven square cavity. **Approach:** The Brinkman-Forcheimer equation is coupled with the lattice Boltzmann formulation to predict the velocity field in the system. Three numerical experiments were preformed with different values of Darcy number to investigate the effect of porosity on the fluid flow. **Results:** In the current study, we found that the magnitude of velocity, strength of vortex and velocity boundary layer is significantly affected porosity of the media. **Conclusion:** The lattice Boltzmann simulation scheme is capable in prediction of fluid flow behavior through porous media.

Key words: Finite difference, lattice Boltzmann, Brinkman-Forcheimer, porosity, lid-driven cavity flow

INTRODUCTION

Numerical and experiment investigations of the fluid flow through porous media are important for a wide range of situation varying from environmental applications to inkjet printing technology. Since the early works by researchers (Darcy, 1856; Forcheimer, 1901; Brinkman, 1947), a great deal of theoretical and experimental researches was dedicated to investigate this phenomenon. The fundamental interest comes from the concern to understand the mass transfer mechanism (Pascal, 1988; Bekri and Adler, 2002, Chan *et al*., 2007) and fluid flow behavior through variable porosity of the media (Davis and Flame, 1996; Joseph *et al*., 2002). On the other hand, a similar interest was provoked by the wide range of engineering applications utilizing this type of phenomenon (Oltean *et al*., 2008; Yang and Hwang, 2009).

In general, the current numerical models to predict the flow through porous media fall in four categories: the Darcy model, the Forcheimer model, the Brinkman model and the Brinkman-Forcheimer model. The Darcy model was developed based on the original Darcy equation. However, this model is reported to produce unsatisfactory results when compared with the theoretical prediction based on Darcy law. In Forcheimer model, it considers the non-linear drag effect due to the solid matrix while the Brinkman model includes the viscous stress introduced by the solid boundary. Even though these two models have been widely used by many researchers, they are not general enough to be applicable for a medium with variable porosity. For this reason, Nithiarasu *et al*. (1997) proposed a generalized non-Darcian porous medium model, a combination of Brinkman and Forcheimer model and produced excellent results for natural convective heat transfer in a fluid saturated variable porosity medium when compared with the experimental data.

In recent years, the Lattice Boltzmann Method (LBM) has received considerable attention as an alternative approach for simulating wide range of fluid flow. Unlike other numerical methods, LBM predicts the evolution of particle distribution function and calculates the macroscopic variables by taking moment to the distribution function. The LBM has a number of advantages over other conventional computational fluid dynamics approaches. The algorithm is simple and can be implemented with a kernel of just a few hundred lines. The algorithm can also be easily modified to allow for the application of other, more complex simulation components. For example, the LBM can be extended to describe the evolution of binary mixtures (Nor Azwadi and Tanahashi, 2006), or extended to allow for more complex boundary conditions (Shan and Chen, 1993). Thus the LBM is an ideal tool in fluid simulation.

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The objective of this study is to couple the lattice Boltzmann formulation with the Brinkman-Forcheimer equation to solve fluid flow through porous media. After showing how the formulation of the particles interactions fit in to the framework of complex porous structure, numerical results of flow through porous media in a square cavity driven by constant lid's velocity are presented to highlight the applicability of the approach.

MATERIALS AND METHODS

Following Nithiarasu *et al*. (1997), the generalized model for incompressible fluid flow through porous media can be expressed by the following equations:

$$\nabla \cdot u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)\left(\frac{u}{\varepsilon}\right) = -\frac{1}{\rho}\nabla(\varepsilon p) + \upsilon_\varepsilon \nabla^2 u + F \tag{2}$$

Where:

υ_ε = The effective viscosity and

ε = The porosity of the medium

F represents the total body force due to the presence of porous media and is given by:

$$F = -\frac{\varepsilon\upsilon}{K}u - \frac{1.75}{\sqrt{150\varepsilon K}}|u|u \tag{3}$$

Where:

υ = The kinematic viscosity and

K = The permeability of which can be related to no dimensional parameter of Darcy number $D\alpha$ as follow:

$$K = Da \times H^2 \tag{4}$$

where, H is the characteristic length.

In the formulation of LBM, the starting point is the evolution equation, discrete in space and time, for a set of distribution functions f. If a two-dimensional nine-velocity model (D2Q9) is used, then the evolution equation for a given f takes the following form:

$$f_i(x + e_i\Delta t, t + \Delta t) - f_i(x,t) = -\frac{1}{\tau}\left[f_i(x,t) - f_i^{eq}(x,t)\right] + F_i \tag{5}$$

Where:

Δt = The time step

e = The particle's velocity

τ = The relaxation time for the collision and i = 0,1,…,8

Note that the term on the right hand side of Eq. 5 is the collision term where the BGK approximation has been applied (Bhatnagar *et al*., 1954). Here, e denotes the discrete velocity set and expressed as:

$$e_i = \begin{cases} (0,0), & i = 0 \\ (\pm 1,0),(0,\pm 1), & i = 1-4 \\ (\pm 1,\pm 1), & i = 5-8 \end{cases} \tag{6}$$

f_i^{eq} is an equilibrium distribution function, the choice of which determines the physics inherent in the simulation. For D2Q9 model, f_i^{eq} is expressed as:

$$f_i^{eq} = \rho\omega_i\left[1 + 3e_i \cdot u + \frac{9}{2}\frac{(e_i \cdot u)^2}{\varepsilon} - \frac{3}{2}\frac{(u)^2}{\varepsilon}\right] \tag{7}$$

where, the weights are ω_0 = 4/9, $\omega_{1\text{-}4}$ = 1/9 and $\omega_{5\text{-}8}$ = 1/36. The time relaxation and the effective viscosity can be related as follow:

$$\upsilon_\varepsilon = \frac{1}{3}\left(\tau - \frac{1}{2}\right) \tag{8}$$

In order to obtain the correct macroscopic governing equations, the forcing term F_i must be expressed in terms of medium porosity as follow:

$$F_i = \rho\omega_i\left(1 - \frac{1}{2\tau}\right)\left[3e_i \cdot F + \frac{9(uF{:}e_ie_i)^2}{\varepsilon} - \frac{3u \cdot F}{\varepsilon}\right] \tag{9}$$

The macroscopic density the macroscopic flow velocity can then be calculated as follow:

$$\rho = \sum_i f_i \tag{10}$$

$$v = \sum_i e_i f_i \tag{11}$$

and:

$$u = \frac{v}{c_0 + \sqrt{c_0^2 + c_1|v|}} \tag{12}$$

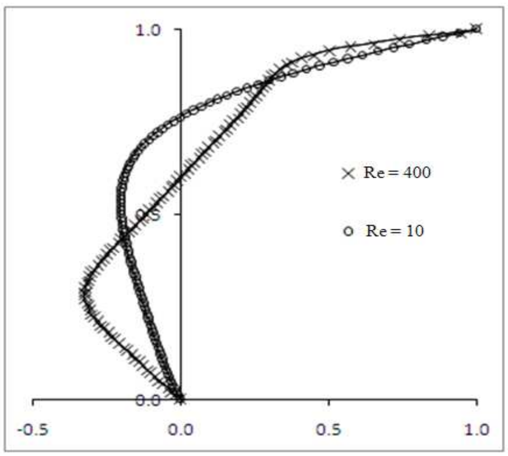
Where:

$c_0 = (1+\varepsilon\upsilon/2K)/2$ and $c_1 = 1.75\varepsilon\big/2\sqrt{150\varepsilon^3 K}$

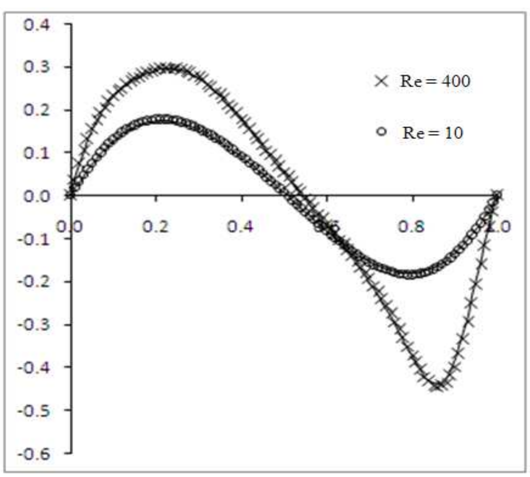
It is noted that, if we set ε = 1, the lattice Boltzmann equation reduces to the standard equation of free fluid flows.

RESULTS

For the sake of code validation, we carried out numerical investigation for the lid-driven cavity flow by setting ε→1. Figure 1 shows the comparisons of plotted velocity profiles along the mid-height and mid-width of the cavity with the finite different solution to the Navier-Stokes equations.



(a)



(b)

Fig. 1: Comparison of (a) horizontal velocity component at mid width of the caivity, (b) vertical velocity component at mid-height of the cavity, between finite different solution to Navier-Stokes equation (solid line) and current method (symbol)

In our next investigation, the value of Reynolds number and medium porosity were maintained at 10 and 0.1 respectively. Three different Darcy number Dα = 10^{-2}, 10^{-3}, 10^{-4} were set up and the simulations were carried out until the system achieved steady state solution. For comparison, the plots of velocity profiles obtained from the solution to the Navier-Stokes by the finite difference are brought and compared with the current method. The plots of comparison are shown in Fig. 2

Finally, Fig. 3 shows the plots of streamline for every simulation condition.

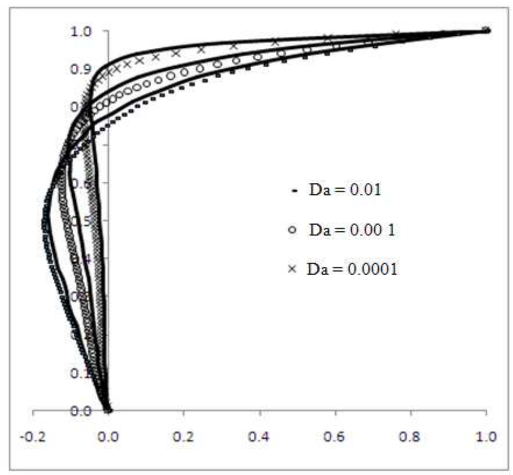
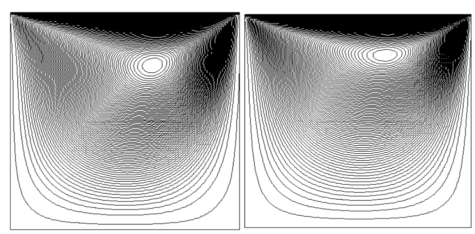
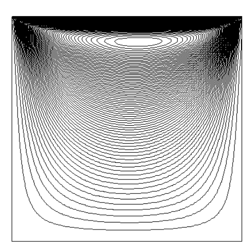


Fig. 2: Comparison of horizontal velocity component at mid-width of the cavity between the current approach (sysmbol) and finite difference formulation (solid line)



(a) (b)



(c)

Fig. 3: Plots of streamlines for (a) Dα = 10^{-2}; (b) Dα = 10^{-3} and (c) Dα = 10^{-4}

DISCUSSION

Figure 1 demonstrates that the current method produces an excellent agreement with the benchmark solution. This gives us confidence to carry out prediction of fluid flow through porous media.

The results in Fig. 2 also clearly demonstrate excellent agreement between these two approaches. Finally, as can be seen from the Fig. 3, the velocity boundary layer becomes thinner and thinner near the top wall as the Dα decrease. The strength of vortex also becomes weaker. All of these findings are in good agreement with the previous studies (Nithiarasu *et al*., 1997; Oltean *et al*., 2008; Yang and Hwang, 2009).

CONCLUSSION

The fluid flow behavior through porous media in a lid-driven square cavity has been studied using the combination of Brinkman-Forcheimer lattice Boltzmann approaches. We found that the present approach correctly predicted the flow feature for different Darcy numbers and gives excellent agreement with the results of previous studies. The results obtained demonstrate that this proposed approach in the lattice Boltzmann model is very efficient procedure to study fluid flow in a lid-driven square cavity with the presence of porous medium. Computation at various values of porosity to investigate the behavior of fluid flow at non-Darcian region will be our near future research topic.

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