

Original Research Paper

# Physical and Geometrical Effects on the Vibratory Pattern of a Pipe Carrying Fluid Flow

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## Article history

Received: 06-07-2020

Revised: 20-10-2020

Accepted: 22-10-2020

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**Abstract:** In this research, the first natural frequencies and critical velocities of a fluid flow-conveying pipe are obtained with finite element method. By discretizing the structure wall and fluid flow, starting from beam type with two degrees of freedom per node. We determine the equation of motion from the fluid-structure coupling using Lagrange energy principle. Parameters frequencies and critical velocity of fluid flow are calculated by using a program developed on MATLAB. The results are compared with those predicted by the differential transformation method. Stability and instability properties of the system are analyzed after calculating the natural frequencies and the fluid critical velocity in terms of various parameters.

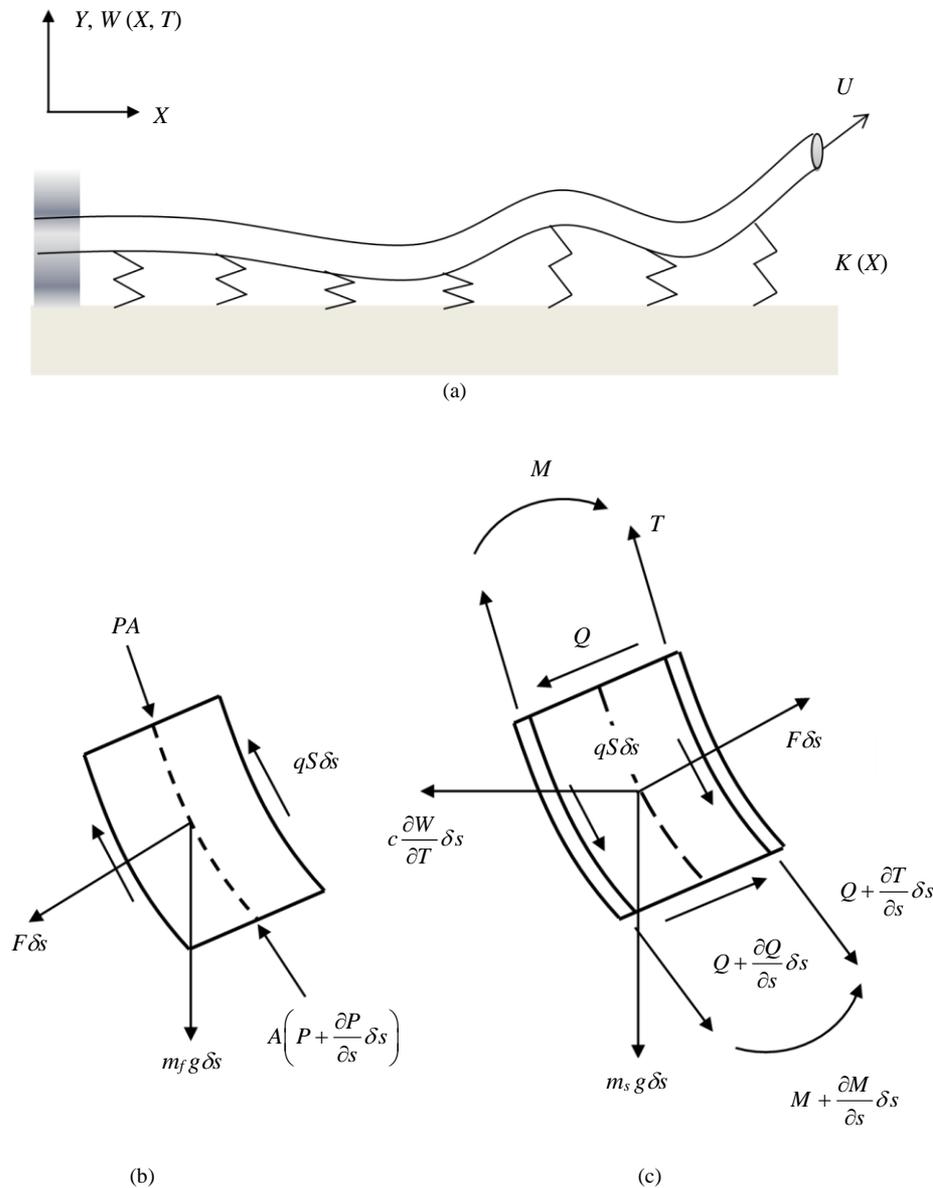
**Keywords:** Fluid-Conveying Pipe, Natural Frequencies, Critical Velocity, Elastic Foundation, Instability, MATLAB

## Introduction

When a structure is contacted with a fluid, the elastic compartment undergoes a significant change; the increase of the natural frequencies observed during interaction of fluid with structure can even affect the system stability. Study of this subject is much ramified, Païdoussis (1970; 1981; 2014; Païdoussis and Besancon, 1981; Païdoussis and Curling, 1985; Païdoussis and Moon, 1988; Païdoussis *et al.*, 2007) is the most famous researcher on the topic of mating and thanks to his research we obtained the linear and non-linear equation of the pipes motion under the influence of many factors. All these researches were gathered with others in a book (Païdoussis, 2014), in which he explores the concept of pipelines stability and instability. Doaré and de Langre (2002a-b; 2000; 2006), he touched better on the issue of instability, we find in his study that he studied instability in its two types (dynamic, static) and that is by calculating the frequencies in terms of the fluid critical velocity. A significant amount of research has been carried out the dynamics of cantilevered pipe-conveying fluid, by means of linear mathematical models (Païdoussis, 2014) and there are other work in the same context, analytical, such as Galerkin method (Chellapilla and Simha, 2007; 2008), semi-analytical,

by DQM in (Qian *et al.*, 2009) and by DTM in (Ni *et al.*, 2011), numerical by finite Element Method (FEM), in (Sadeghi and Karimi-Dona, 2011; Mostafa, 2014; Dahmane *et al.*, 2016). There are many contemporary studies dealing with the subject of pipe vibration in term of fluid flow under physical and geometrical effects (Ghayesh *et al.*, 2018a-b; Jiya *et al.*, 2018; Zhang *et al.*, 2018; Liang *et al.*, 2018; Wang *et al.*, 2018; Liu *et al.*, 2018; Sazesh and Shams, 2019). All these studies did not take into consideration the concept of the range (field) instability as well as the analysis and the instability parameters, which constitutes an obstacle and challenge for engineers, especially in the heat exchangers as nuclear production, industry pipelines, fuel pipes in high duty engines, hydropower systems and solution mining applications.

In the present study, calculation methods have been developed for the free vibrations analysis of cantilevered pipe conveying fluid. The numerical methods were developed, modeling of structure (pipe)-incompressible fluid and was conducted by the standard finite element method, using beam type with two degrees of freedom per node. The proper frequencies and critical velocity of the system are calculated using a program developed on MATLAB. This allows us to study instability and discover its most important characteristics.



**Fig. 1:**(a) Cantilevered pipe-conveying fluid resting on a foundation elastic Winkler-type, (b) forces on fluid element; (c) forces and moments on pipe element  $\delta s$  (Païdoussis, 2014)

### Differential Equation of Motion

The problem to be considered is the vibration analysis of a fluid conveying pipe system on an elastic foundation Winkler-model. The derivation of the equation is based on bernoulli-euler elementary beam theory. The physical model of system is shown in Fig. 1a and 1b shows forces on fluid element, Fig. 1c shows forces and pipe element moment.

The equation for conveying pipe carrying fluid on a Winkler elastic foundation is given as (Dahmane *et al.*, 2016):

$$EI \frac{\partial^4 Y}{\partial X^4} + m_f U^2 \frac{\partial^2 Y}{\partial X^2} + 2m_f U \frac{\partial^2 Y}{\partial X \partial T} + (m_s + m_f) \frac{\partial^2 Y}{\partial T^2} + KY = 0 \quad (1)$$

where, the pipe is long and straight  $L$  conveying an incompressible fluid with steady speed  $U$ ; the motions are small  $\delta s$ . The elastic foundation Winkler-model is  $KY$ ,  $m_s$  and  $m_f$  the masses per unit length of the pipe and the fluid, respectively (Fig. 2). The Boundary conditions are:

$$Y|_{x=0} = \frac{\partial Y}{\partial X}|_{x=0} = \frac{\partial^2 Y}{\partial X^2}|_{x=L} = \frac{\partial^3 Y}{\partial X^3}|_{x=L} = 0 \quad (2)$$

$$y|_{x=0} = \frac{\partial y}{\partial x}|_{x=0} = \frac{\partial^2 y}{\partial x^2}|_{x=1} = \frac{\partial^3 y}{\partial x^3}|_{x=1} = 0 \quad (3)$$

where, the non-dimensional parameters (Païdoussis, 2014), we obtain:

$$x = X / Y, y = Y / L, t = \left( EI / (m_f + m_s) \right)^{1/2} T / L^2,$$

$$\beta = m_f / (m_f + m_s),$$

$$u = UL(m_f / EI)^{1/2}, k = KL / EI$$

where, the potential (deformation) energy and the kinetic energy of the solid element expressed by:

$$V_1 = \frac{1}{2} \int_0^L EI \left( \frac{d^2 W}{dX^2} \right)^2 dX \quad (4)$$

$$E_{Cs} = \frac{1}{2} \int_0^L m_s \frac{d^2 W}{dT^2} dX \quad (5)$$

The kinetic energy of the fluid element can be expressed by (Sadeghi and Karimi-Dona, 2011):

$$E_{Cf} = \frac{1}{2} \int m_f \left( U \frac{dW}{dX} + \frac{dW}{dT} \right)^2 dX \quad (6)$$

The potential energy over the length of elastic foundation can be expressed by (Ni *et al.*, 2011; Sadeghi and Karimi-Dona, 2011):

$$V_2 = \frac{1}{2} \int_0^L KW^2 dX \quad (7)$$

The equation of element deflection for straight two dimensional beam element could have the form (Rao, 2011):

$$W(X, T) = \sum_{i=1}^N N_i(X) W_i(T) \quad (8)$$

where,  $[N_i]$  represent the shape function and  $W_i(T)$  is the function which represents the shape of the displacements and rotations at nodes (Fig. 3).

Therefore, Equation (8) becomes:

$$W(X, T) = N_1(X) W_1(T) + N_2(X) \theta_1(T) + N_3(X) W_2(T) + N_4(X) \theta_2(T) \quad (9)$$

Forming shape functions:

$$\begin{cases} N_1 = 1 - 3x^2 + 2x^3 \\ N_2 = L(x - 2x^2 + 3x^3) \\ N_3 = 3x^2 - 2x^3 \\ N_4 = Lx^2(x - 1) \end{cases} \quad (10)$$

where,  $0 \leq x \leq 1$  (Fig. 4).

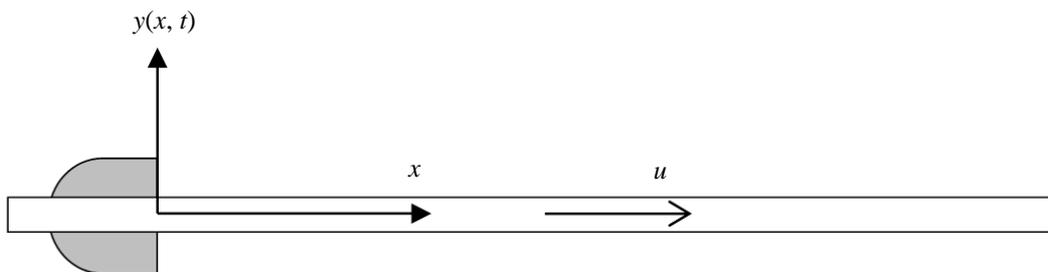


Fig. 2: Clamped-free Pipe with fluid



Fig. 3: Beam element nodal displacements; where,  $W_1, \theta_1, W_2$  et  $\theta_2$ : Are the displacements and rotations at the nodes.

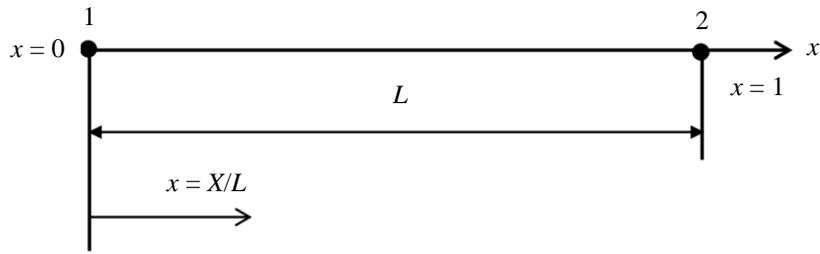


Fig. 4: Reference element

*Lagrange's Principle*

$$\frac{d}{dT} \left( \frac{\partial E_c}{\partial \dot{q}_i} \right) - \frac{\partial E_c}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0 \tag{11}$$

The standard equation of motion in the finite element form is:

$$[M]\{\ddot{q}\} + (C)\{\dot{q}\} + ([K])\{q\} = 0 \tag{12}$$

Where:

$[M] = [M_s] + [M_f]$ : Elementary mass matrix of the system

$[C]$ : Elementary damping matrix of the system

$[K] = [K_s] - [K_f]$ : Elementary stiffness matrix of the system

Considering the displacement vector as:

$$\{Q\} = \{E\} \cdot \exp(\lambda t) \tag{13}$$

$\lambda$  is eigenvalues of the system and the  $\{E\}$  corresponding eigenvectors of this value.

The solution of Equation (12) is very complicated with presence of damping, so we use the variable change method (state-space):

$$E\dot{z} + Gz = 0 \tag{14}$$

where the state variable is:

$$z = \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} \tag{15}$$

The matrices  $[E]$  and  $[G]$  are calculated through variable-change as the following:

$$E = \begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \tag{16}$$

$$G = \begin{bmatrix} C & K \\ -K & 0 \end{bmatrix} \tag{17}$$

And the solution of equation is sought in the general form:

$$z = \begin{Bmatrix} \lambda \{E\} \\ \{E\} \end{Bmatrix} \exp(\lambda t) = \{\tilde{E}\} \exp(\lambda t) \tag{18}$$

The system equation of government can be transformed from state space by:

$$\left\{ \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} - \lambda \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \right\} \begin{Bmatrix} \lambda \{E\} \\ \{E\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{19}$$

$I$  is a unity matrix.

We ask ourselves:

$$H = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{20}$$

Eigenvalues are complex; they give in the form:

$$\lambda^m = \text{Re}^m + j\omega^m \tag{21}$$

Where:

Re: The real part of the eigenvalue and is the damping of our system

$\omega$ : The imaginary part of the eigenvalue, is therefore the proper pulsation of system

The stability and instability of the system under consideration is determined by the sign of real part and the natural frequencies values (the imaginary part) of the complex Eigen-value.

**Results and Discussion**

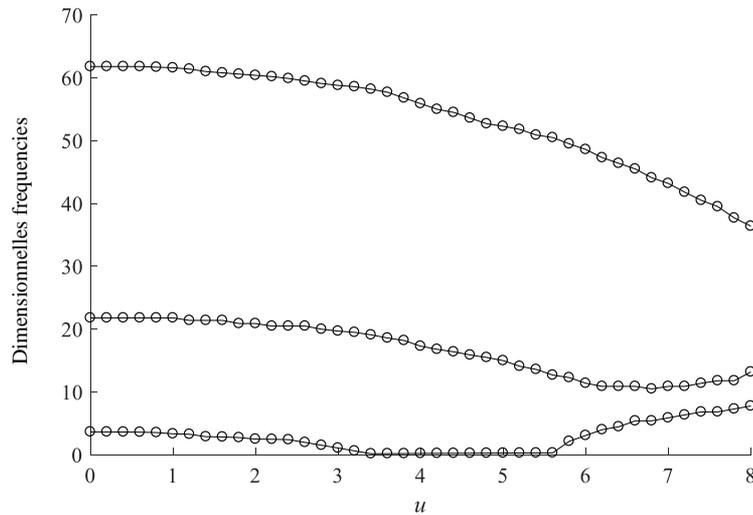
In studying and analyzing the concept of instability, the fundamental natural frequency of a pipe decreases with increasing fluid velocity. There are certain cases where a decrease in this natural frequency can be very important and with large fluid velocities, the pipe may become unstable. Results will be discussed for various values of  $\beta$ , length  $L$ , elastic foundation  $k$  (Winkler type) and the instability parameters for clamped-free pipe. The

elastic modulus of pipe is ( $E = 207$  GPa), pipe length is ( $L = 1 \div 2$  m), fluid density is ( $m_f = 1000$  kg/m<sup>3</sup>), pipe density is ( $m_s = 7850$  kg/m<sup>3</sup>), pipe thickness for ( $\beta = 0.1 \div 0.7$ ) and the outer diameter of the pipe is (0.03 m). Frequencies results are displayed in terms of flow velocity and several parameters by figures, which come in three cases as follows: Mass ratio effect, length effect and elastic foundation effect.

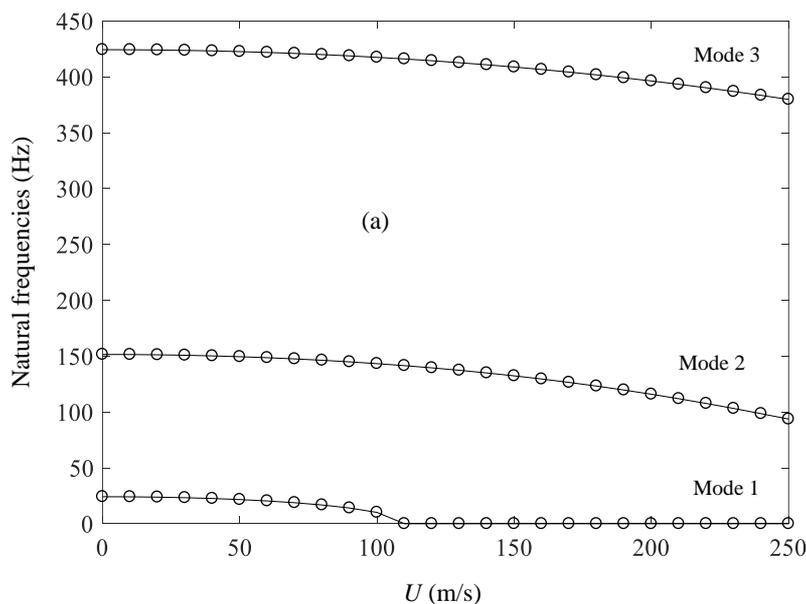
**Mass Ratio Effect**

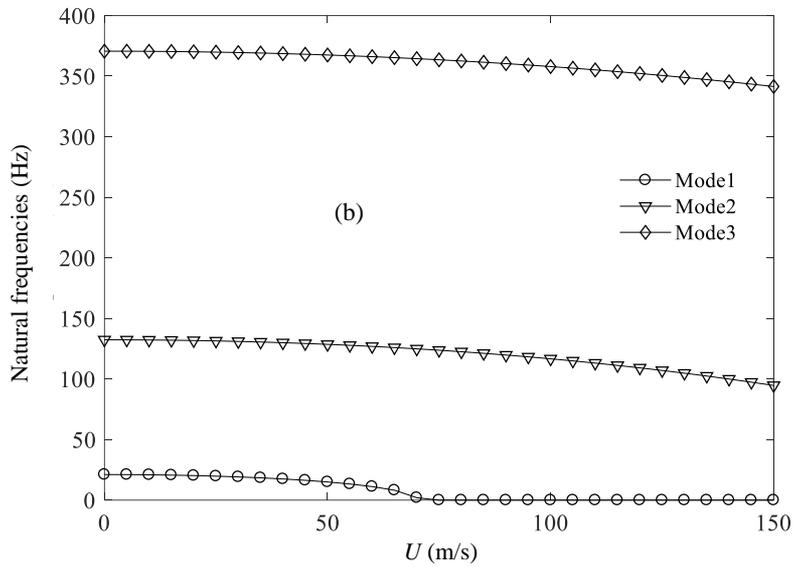
We calculated the natural frequency for first Eigen modes of clamped-free pipe for mass ratios, according to the fluid speed, Fig. 5 and 6.

Firstly, the program has been validated with reference (Ni *et al.*, 2011), by calculating the first three dimensionless frequency as a function of the fluid velocity for cantilevered pipe with internal flow, where  $\beta = 0.5$ . The Fig. 5 shows that the results obtained numerically are similar to those obtained by the analytical approach. The rigidity of a fixed-free beam system is very low than the other boundary conditions, it is preferable to use a dimensioning by calculating the natural frequencies. The Fig. 6 shows that the free vibration of the clamped-free pipe under an internal flow is very sensitive to the variation of mass ratios.

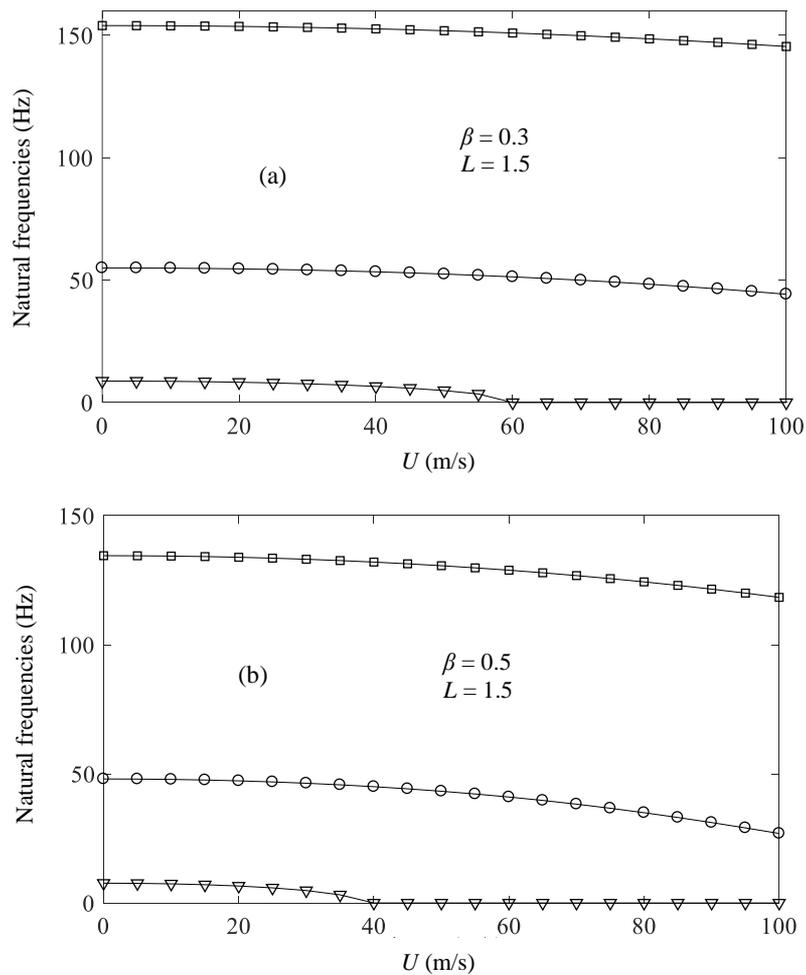


**Fig. 5:** The imaginary component of the dimensionless frequency as a various values of fluid velocity ( $u$ ), for the lowest three modes of a clamped-free pipe conveying fluid, comparison DTM (Ni *et al.*, 2011) (ooo) and FEM (—),  $\beta = 0.5$

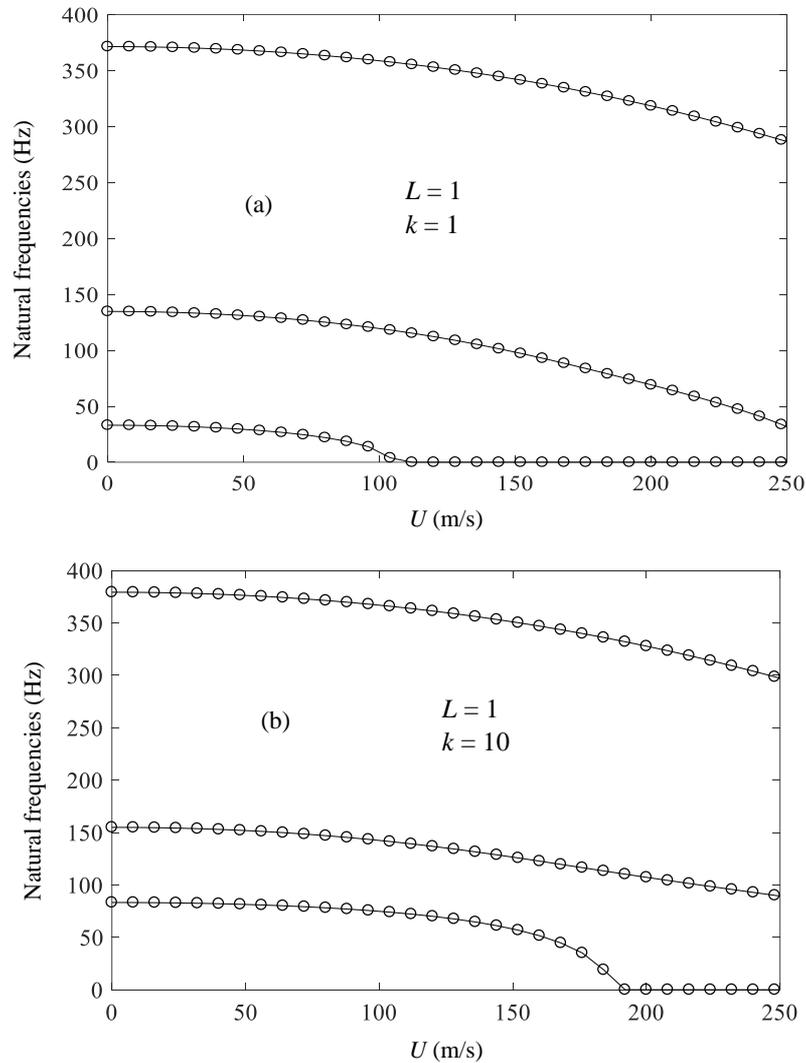




**Fig. 6:** Naturel frequency (Hz) for three proper modes on fluid velocity function of cantilevered pipe conveying fluid,  $L = 1$ , (a)  $\beta = 0.3$ , (b),  $\beta = 0.5$



**Fig. 7:** Effect of length and mass ratio on the natural frequency of the cantilevered pipe at different fluid velocities



**Fig. 8:** Effect of foundation stiffness on the natural frequency of the clamped-free pipe at different fluid velocities, (a)  $k = 1$ , (b)  $k = 10$ ,  $\beta = 0.5$

### Length Effect

We calculated the frequency of the first three Eigenmodes clamped-free pipe for two mass ratios, according to the fluid speed where  $L = 1.5$  m, see the Fig. 7.

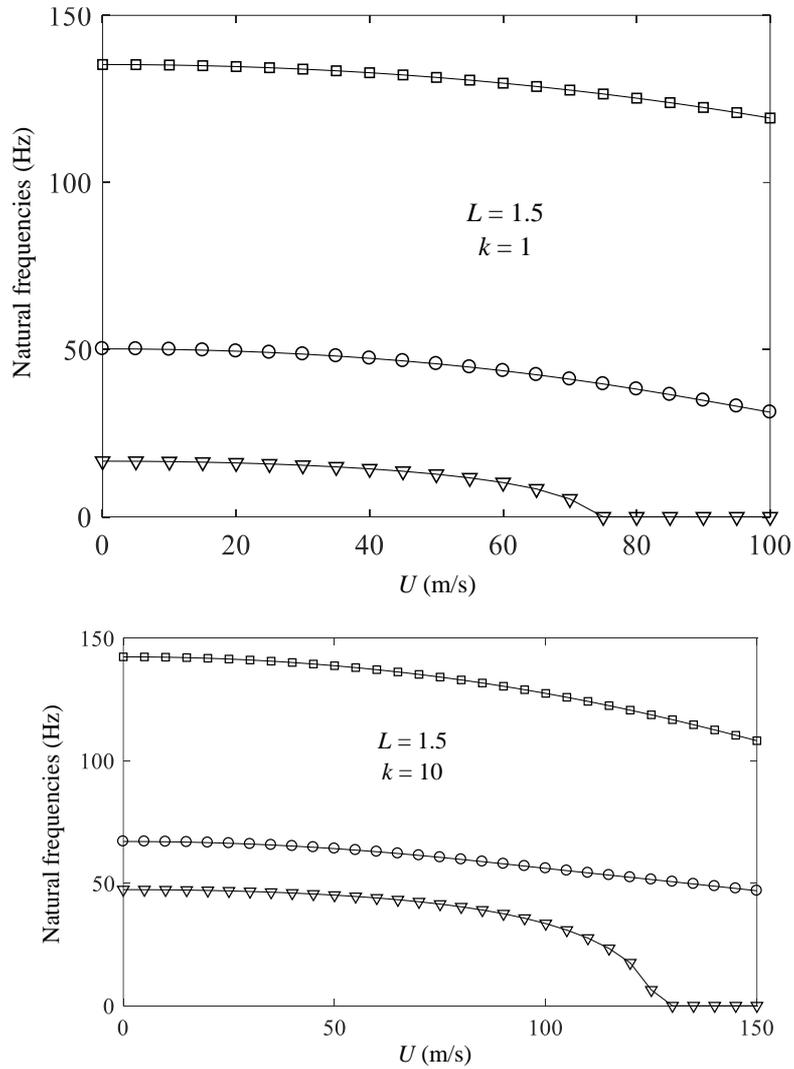
We see in the Fig. 6a that the first frequency is 25 Hz, in the case of a fluid at rest (no flow) for a mass ratio 0.3 and for  $\beta = 0.5$  the first frequency is 22 Hz, Fig. 6b. the same remark for the second frequency which passes from 140 Hz to 150 Hz, however for the third mode, the frequency passes from for 375 Hz for  $\beta = 0.5$  to 425 Hz for  $\beta = 0.3$ . In this case the pipe comparable to a beam ( $U = 0$ ) i.e., all its natural frequencies are imaginary.

### Foundation Effect

We calculated the frequency of the first three Eigenmodes clamped-free pipe on an elastic foundation with

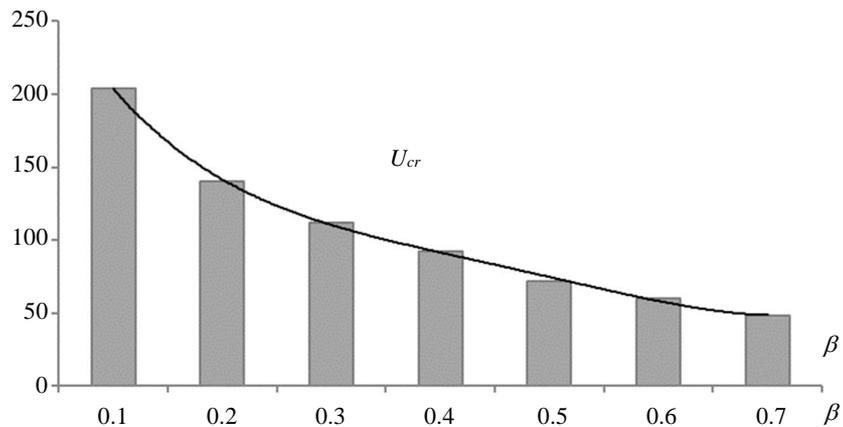
two low values, according to the fluid velocity, for two lengths, Fig. 8 ( $L = 1$  m), Fig. 9 ( $L = 1.5$  m).

In the case of the flowing fluid, the first eigen modes are not similar for different mass ratio. The critical velocity is 76 m/s for  $\beta = 0.5$  and for  $\beta = 0.3$ , the critical speed is 110 m/s. when one increase the velocity, one observes first of all a stabilization of the system and all the natural frequencies see their imaginary part decreasing at the critical velocity of instability. We are now interested in the stability conditions of such system, we see in the Fig. 6 ( $L = 1$ ) and the Fig. 7 that the increase in the length quickly destabilizes the system which depends on  $\beta$ . The effect of the elastic foundation characterized by ( $k$ ), the variation of the first modes of pipe under an internal flow is given by the Fig. 8 and Fig. 9 for two lengths, where mass ratio is 0.5. So the figures show double effect of the foundation and the length.

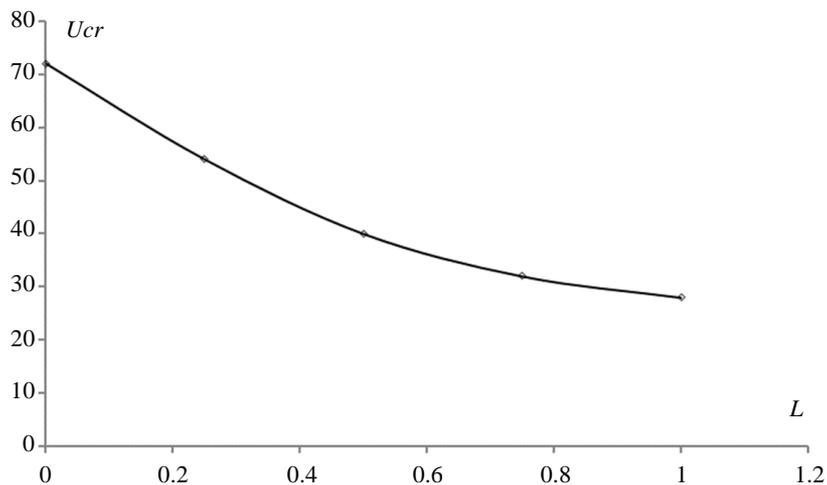


**Fig. 9:** Effect of foundation stiffness on the natural frequency of the clamped-free pipe at different fluid velocities,  $L = 1.5, \beta = 0.5$

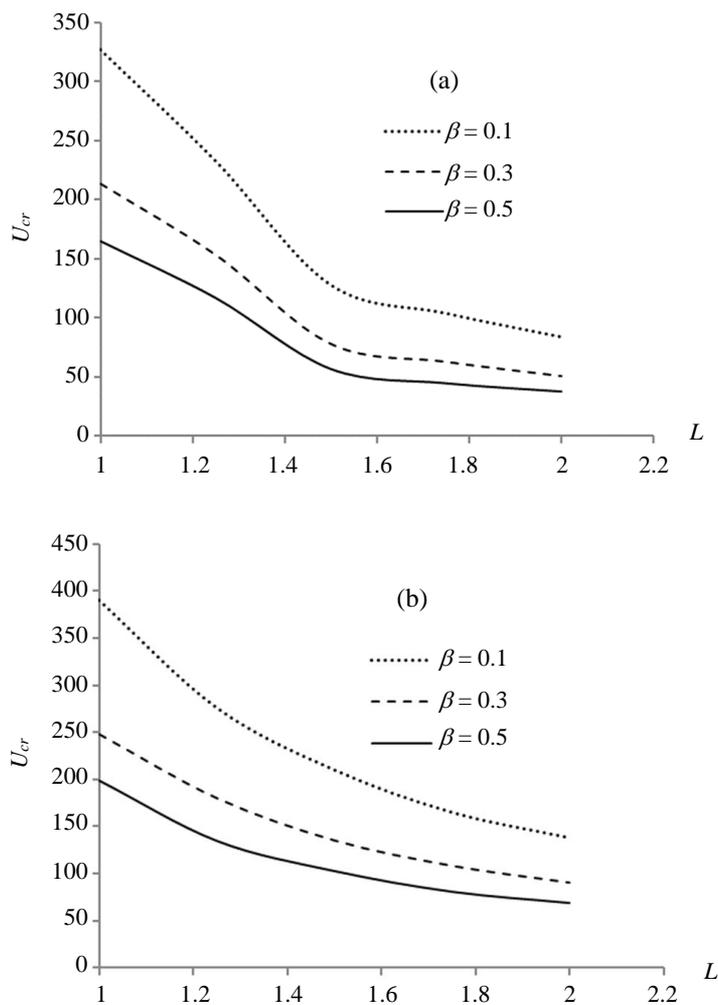
*Pipe Instability Parameters*



**Fig. 10:** The evaluation of the critical speed of instability as a function of the mass ratio for a reference length  $L = 1$



**Fig. 11:** Curve of the instability of the embedded-free pipe, highlighting the destabilizing effect of the length of the system



**Fig. 12:** Curves of the instability of the embedded-free pipe for different elastic foundations, highlighting the destabilizing effect of the length and the destabilizing effect of the mass ratio, (a)  $k = 1$ , (b)  $k = 10$

By studying dynamics (Flutter) where the critical velocity varies according to the masse ratio in interval  $[0.1 \div 0.7]$  (Fig. 10), we find that the variation curve takes a hyperbolic form. The Fig. 11 shows the variation of the critical velocity as a function of the length; we note that this curve has the same shape as the curve previous figure. We found here that the parameter  $\beta$  has a destabilizing effect. Critical velocity is very sensitive to variation in length. The Fig. 12 represents the influence of the length on the variation of the critical velocity for different stiffness with different mass ratio. For  $k = 1$ , we find that the slopes straighten around  $L = 1.5$ . We repeated the same work for  $k = 10$ , the curve have the same gaits.

## Conclusion

The numerical aspect gives solutions in complex planes by determining the eigen-modes, an imaginary part to present the eigen-frequencies according to the fluid velocity for various parameters. The first observation that we can make that the natural frequencies of the system depends on the fluid velocity. We observe that instability appears when the velocity exceeds a threshold called the critical instability velocity. The critical velocity of the fluid varies according to the mass ratio and translates the stability of our system, while the foundation increases the system rigidity and consequently the natural frequencies and the critical speed. It has been observed that there is an absence of static buckling instability.

## Author's Contributions

**Dahmane Mouloud and Samir Zahaf:** The authors of the idea and the research, conduced the theoretical study.

**Mawhoub Soubih and Djilali Boutchicha:** Extracted elementary matrix using the finite element method.

**Benkhattab Mohamed:** Undertook programming using the Matlab code.

## Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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